Static properties and semileptonic decays of doubly heavy baryons in a nonrelativistic quark model.

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We evaluate static properties and semileptonic decays for the ground state of doubly heavy Ξ, Ξ', Ξ^* and Ω, Ω', Ω^* baryons. Working in the framework of a nonrelativistic quark model, we solve the three–body problem by means of a variational ansazt made possible by heavy quark spin symmetry constraints. To check the dependence of our results on the inter-quark interaction we use five different quark-quark potentials that include a confining term plus Coulomb and hyperfine terms coming from one–gluon exchange. Our results for static properties (masses, charge and mass radii, magnetic moments. . .) are, with a few exceptions for the magnetic moments, in good agreement with a previous Faddeev calculation. Our much simpler wave functions are used to evaluate semileptonic decays of doubly heavy $\Xi, \Xi'(J = 1/2)$ and $\Omega, \Omega'(J = 1/2)$ baryons. Our results for the decay widths are in good agreement with calculations done within a relativistic quark model in the quark–diquark approximation.

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I. INTRODUCTION

Even though only recently the mass of a doubly heavy baryon has been measured experimentally [1], the subject has been attracting attention for a long time. Magnetic moments of doubly charmed baryons were evaluated back in the 70's by Lichtenberg [2] within a nonrelativistic approach. The infinite heavy quark mass limit was already used in the 90's to relate the spectrum of doubly heavy baryons to the one of mesons with a single heavy quark [3], or to analyze their semileptonic decay [4]. In hadrons with a heavy quark and working in the infinite heavy quark mass limit the dynamics of the light degrees of freedom becomes independent of the heavy quark flavor and spin. This is known as heavy quark symmetry (HQS) [5, 6, 7, 8]. This symmetry was developed into an effective theory (HQET) [9] that allowed a systematic, order by order, evaluation of corrections in inverse powers of the heavy quark masses. Unfortunately ordinary HQS can not be applied directly to hadrons containing two heavy quarks as the kinetic energy term needed in those systems to regulate infrared divergences breaks heavy flavor symmetry [10]. For those

Baryon	S	J^P	I	S_h^π	Quark content
Ξ_{cc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ξ^*_{cc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	ccl
Ω_{cc}	-1	$\frac{1}{2}^+$	0	1^+	ccs
Ω^*_{cc}	-1	$\frac{3}{2}^+$	0	1^+	ccs
Ξ_{bb}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ξ^*_{bb}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bbl
Ω_{bb}	-1	$\frac{1}{2}^+$	0	1^+	bbs
Ω^*_{bb}	-1	$\frac{3}{2}^+$	0	1^+	bbs
Ξ'_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	bcl
Ξ_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ξ^*_{bc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bcl
Ω'_{bc}	-1	$\frac{1}{2}^+$	0	0^+	bcs
Ω_{bc}	-1	$\frac{1}{2}^+$	0	1^+	bcs
Ω^*_{bc}	-1	$\frac{3}{2}^+$	0	1^+	bcs

TABLE I: Quantum numbers of doubly heavy baryons analyzed in this study. S, J^P are strangeness and the spin parity of the baryon, I is the isospin, and S_h^π is the spin parity of the heavy degrees of freedom. l denotes a light u or d quark .

hadrons the symmetry that survives is heavy quark spin symmetry (HQSS) [11], which amounts to the decoupling of the heavy quark spins in the infinite heavy quark mass limit. In that limit one can consider the total spin of the two heavy quark subsystem (S_h) to be well defined. In this work we shall assume this is a good approximation for the actual heavy quark masses. This approximation, which is the only one related to the infinite heavy quark mass limit that we shall use, will certainly simplify the solution of the baryon three–quark problem.

Solving the three–body problem is not an easy task and here we shall do it by means of a variational approach. The approach, with obvious changes, was already applied with good results in the study of baryons with one heavy quark [12]. This method, that leads to simple and manageable wave functions, is made possible by the simplifications introduced in the problem by the fact that we can consider S_h to be well defined. We shall consider several simple phenomenological quark–quark interactions [13, 14, 15] which free parameters have been adjusted in the meson sector and are thus free of three–body ambiguities. The use of different interactions will allow us to estimate part of the theoretical uncertainties affecting our calculation.

Our simple variational calculation reproduces the results for static properties obtained in Ref. [15] by solving more involved Faddeev type equations. Static properties like masses and magnetic moments of doubly heavy baryons have also been studied in other models. Masses have been calculated in the relativistic quark model assuming a light quark heavy diquark structure [16], the potential approach and sum rules of QCD [17], the nonperturbative QCD approach [18], the Bethe–Salpeter equation applied to the light quark heavy diquark [19], the nonrelativistic quark model with harmonic oscillator potential [20] or with the use of QCD derived potentials [21, 22], the relativistic quasi–potential quark model [23], with the use of the Feynman-Hellman theorem and semi-empirical mass formulas [24], or in HQET [25]. There are also lattice determinations by the UKQCD Collaboration [26]. Similarly, magnetic moments have been evaluated in a nonrelativistic approach [2], in the relativistic three–quark model [27], the relativistic quark model using different forms of the relativistic kinematics [28], in the skyrmion model [29], in the Dirac equation formalism [30], or using the MIT bag model [31].

We shall further use our manageable wave functions to study semileptonic decays of doubly, $J = 1/2$, baryons. We shall evaluate form factors, decay widths and angular asymmetry parameters. Previous calculations of semileptonic decay widths have been done in different relativistic quark model approaches [32, 33, 34], or with the use of HQET [35].

The paper is organized as follows. In Sect. II we study the Hamiltonian of the system (Subsect. II A) and briefly introduce the different inter-quark interactions used in this work (Subsect. II B). The variational wave functions are discussed in Sect. III. In Sect. IV we present results for the static properties: masses (Subsect. IV A), charge and mass densities and radii (Subsect. IV B), and magnetic moments (Subsect. IV C). Semileptonic decays are analyzed in Sect. V. After the presentation of general formulas, in Subsect. V A we relate the form factors to matrix elements and show how the latter ones are evaluated within our model. In Subsect. V B we present our results for the form factors, differential and total semileptonic decay widths, and angular asymmetry parameters. The findings of this work are summarized in Sect. VI. The paper also includes three appendices: in appendix A we give the variational wave function parameters for the five different inter–quark interactions used. In appendix B we relate the form factors for semileptonic decay to the two basic integrals in terms of which all of them can be obtained. Finally, in appendix C we give explicit expressions for those basic integrals.

In Table I we summarize the quantum numbers of the doubly heavy baryons considered in this study.

II. THREE BODY PROBLEM

A. Intrinsic Hamiltonian

In the Laboratory (LAB) frame (see Fig. 1), the Hamiltonian (H) of the three quark (h_1, h_2, q, where h_1, $h_2 = c, b$ and $q = l(u,\, d)$, s) system reads:

$$H = \sum_{j=h_1,\, h_2,\, q} \left(m_j - \frac{\vec{\nabla}^2_{\vec{x}_j}}{2m_j} \right) + V_{h_1 h_2} + V_{h_1 q} + V_{h_2 q} \tag{1}$$

where m_{h_1}, m_{h_2}, m_q are the quark masses and the quark-quark interaction terms V_{jk} depend on the quark spin-flavor quantum numbers and the quark coordinates ($\vec{x}_{h_1}, \vec{x}_{h_2}$, $\vec{x}_q$ for the h_1, h_2, q quarks respectively). To separate the Center of Mass (CM) free motion, we go to the light quark frame ($\vec{R}, \vec{r}_1, \vec{r}_2$),

$$\begin{aligned} \vec{R} &= \frac{m_{h_1}\vec{x}_{h_1} + m_{h_2}\vec{x}_{h_2} + m_q\vec{x}_q}{m_{h_1} + m_{h_2} + m_q} \\ \vec{r}_1 &= \vec{x}_{h_1} - \vec{x}_q \\ \vec{r}_2 &= \vec{x}_{h_2} - \vec{x}_q \end{aligned} \tag{2}$$

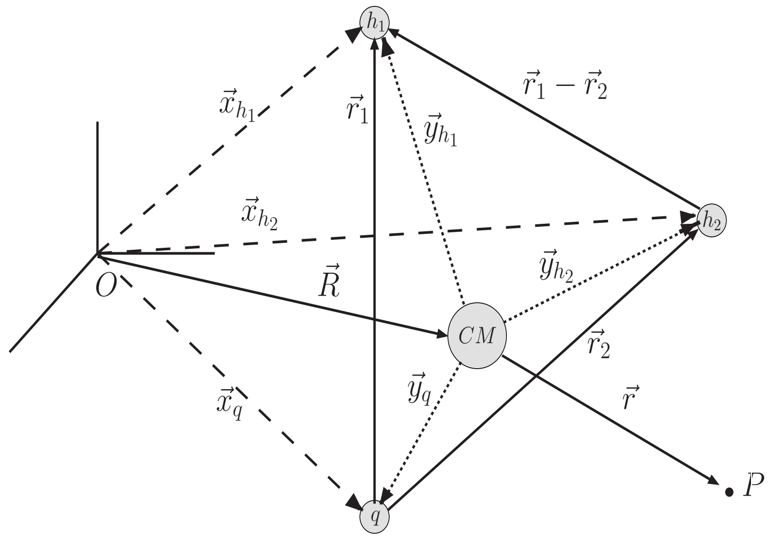

FIG. 1: Definition of different coordinates used through this work.

where $\vec{R}$ and $\vec{r}_1$, $\vec{r}_2$ are the CM position in the LAB frame and the relative positions of the h_1, h_2 heavy quarks with respect to the light quark q. The Hamiltonian now reads

$$H = -\frac{\vec{\nabla}^2_{\vec{R}}}{2\overline{M}} + H^{\rm int} \tag{3}$$

$$\begin{aligned} H^{\rm int} &= \sum_{j=1,2} H_j^{sp} + V_{h_1h_2}(\vec{r}_1 - \vec{r}_2, \, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_q} + \overline{M} \\ H_j^{sp} &= -\frac{\vec{\nabla}_j^2}{2\mu_j} + V_{h_jq}(\vec{r}_j, \, spin), \;\; j = 1, 2 \end{aligned} \tag{4}$$

where $\overline{M} = m_{h_1} + m_{h_2} + m_q$, $\mu_j = \left(1/m_{h_j} + 1/m_q\right)^{-1}$ and $\vec{\nabla}_j = \partial/\partial_{\vec{r}_j}$, $j = 1, 2$. The intrinsic Hamiltonian $H^{\rm int}$ describes the dynamics of the baryon. It consists of the sum of two single particle Hamiltonian H_j^{sp}, each of them describing the dynamics of a heavy-light quark system, plus the heavy–heavy interaction term, including the Hughes-Eckart term ($\vec{\nabla}_1 \cdot \vec{\nabla}_2$), and the sum of the quark masses $\overline{M}$. We will use a variational approach to solve it.

B. Quark–Quark Interactions

We have examined five different interactions, one suggested by Bhaduri and collaborators [13] (BD) and four suggested by B. Silvestre-Brac and C. Semay [14, 15] (AL1, AL2, AP1 y AP2). All of them contain a confinement term, plus Coulomb and hyperfine terms coming from one-gluon exchange, and differ from each other in the form factors used for the hyperfine terms, the power of the confining term or the use of a form factor in the one gluon exchange Coulomb potential. All free parameters in the potentials had been adjusted to reproduce the light (π, ρ, K, K^*, etc.) and heavy-light (D, D^*, B, B^*, etc.) meson spectra[1]. All details on the above interactions can be found in Refs. [13, 14, 15].

These interactions were also used in Ref. [15] to obtain, within a Faddeev calculation, the spectrum and static properties of heavy baryons. Our simpler variational method will not only give equally good results for the observables analyzed in [15], but it will also provide us with easy to handle wave functions that can be used to evaluate other observables.

[1] To get the quark–quark interaction starting from a quark-antiquark one the usual $V_{ij}^{qq} = V_{ij}^{q\bar{q}}/2$ prescription, coming from a $\vec{\lambda}_i\vec{\lambda}_j$ color dependence ($\vec{\lambda}$ are the Gell-Mann matrices) of the whole potential, has been assumed.

III. VARIATIONAL WAVE FUNCTIONS

For the above mentioned interactions, we have that both the total spin and the internal orbital angular momentum given as

$$\begin{aligned}\vec{S} &= (\vec{\sigma}_{h_1} + \vec{\sigma}_{h_2} + \vec{\sigma}_q)/2 \\ \vec{L} &= \vec{l}_1 + \vec{l}_2, \qquad \text{with } \vec{l}_j = -\mathrm{i}\ \vec{r}_j \times \vec{\nabla}_j, \quad j = 1,2 \end{aligned} \tag{5}$$

commute with the intrinsic Hamiltonian and are thus well defined. We are interested in the ground state of baryons with total angular momentum $J = 1/2,\, 3/2$ so that we can assume the orbital angular momentum of the baryons to be $L = 0$. This implies that the spatial wave function can only depend on the relative distances r_1, r_2 and $r_{12} = |\vec{r}_1 - \vec{r}_2|$. Furthermore when the heavy quark mass is infinity ($m_h \to \infty$), the total spin of the heavy degrees of freedom, $\vec{S}_{\text{heavy}} = (\vec{\sigma}_{h_1} + \vec{\sigma}_{h_2})/2$, commutes with the intrinsic Hamiltonian, since the spin–spin terms in the potentials vanish in this limit. We can then assume the spin of the heavy degrees of freedom to be well defined.

With these simplifications we have used the following intrinsic wave functions in our variational approach[2]

- $\Xi_{h_1h_2}$, $\Omega_{h_1h_2}$*−type baryons:*

$$\begin{aligned}|\Xi_{h_1h_2},\, \Omega_{h_1h_2}; J = \frac{1}{2}, M_J\rangle &= \sum_{M_{S_h} M_{S_q}} (1\frac{1}{2}\frac{1}{2}|M_{S_h} M_{S_q} M_J)\ |h_1h_2; 1 M_{S_h}\rangle \otimes |q; \frac{1}{2} M_{S_q}\rangle \\ &\qquad \times \Psi^{\Xi,\,\Omega}_{h_1h_2}(r_1, r_2, r_{12}) \end{aligned} \tag{6}$$

  where M_J is the third component of the baryon total angular momentum while $|h_1h_2; S_h, M_{S_h}\rangle$ and $|q; \frac{1}{2} M_{S_q}\rangle$ represent spin states of the h_1h_2 subsystem and the light quark respectively. $(j_1 j_2 j|m_1 m_2 m)$ is a Clebsch-Gordan coefficient. For $h_1 = h_2$ we need $\Psi^{\Xi,\,\Omega}_{h_1h_1}(r_1, r_2, r_{12}) = \Psi^{\Xi,\,\Omega}_{h_1h_1}(r_2, r_1, r_{12})$ to guarantee a complete symmetry of the wave function under the exchange of the two heavy quarks.

- $\Xi^*_{h_1h_2}$, $\Omega^*_{h_1h_2}$*−type baryons:*

$$\begin{aligned}|\Xi^*_{h_1h_2},\, \Omega^*_{h_1h_2}; J = \frac{3}{2}, M_J\rangle &= \sum_{M_{S_h} M_{S_q}} (1\frac{1}{2}\frac{3}{2}|M_{S_h} M_{S_q} M_J)\ |h_1h_2; 1 M_{S_h}\rangle \otimes |q; \frac{1}{2} M_{S_q}\rangle \\ &\qquad \times \Psi^{\Xi^*,\,\Omega^*}_{h_1h_2}(r_1, r_2, r_{12}) \end{aligned} \tag{7}$$

  Similarly to the case before for $h_1 = h_2$ we need $\Psi^{\Xi^*,\,\Omega^*}_{h_1h_1}(r_1, r_2, r_{12}) = \Psi^{\Xi^*,\,\Omega^*}_{h_1h_1}(r_2, r_1, r_{12})$.

- $\Xi'_{h_1h_2}$, $\Omega'_{h_1h_2}$*−type baryons:*

$$\begin{aligned}|\Xi'_{h_1h_2},\, \Omega'_{h_1h_2}; J = \frac{1}{2}, M_J\rangle &= |h_1h_2; 00\rangle \otimes |q; \frac{1}{2} M_J\rangle \\ &\qquad \times \Psi^{\Xi',\,\Omega'}_{h_1h_2}(r_1, r_2, r_{12}) \end{aligned} \tag{8}$$

  In this case $h_1 \neq h_2$ and we do not need the orbital part to have a definite symmetry under the exchange of the two quarks.

The spatial wave functions $\Psi(r_1, r_2, r_{12})$ in the above expressions will be determined by the variational principle: $\delta\langle B|H^{\text{int}}|B\rangle = 0$. For simplicity, we shall assume a Jastrow–type functional form[3]:

$$\Psi^B_{h_1h_2}(r_1, r_2, r_{12}) = N\, F^B(r_{12})\, \phi_{h_1q}(r_1)\, \phi_{h_2q}(r_2) \tag{9}$$

where N is a constant, which is determined from normalization

$$1 = \int d^3r_1 \int d^3r_2\ \left|\Psi^B_{h_1h_2}(r_1, r_2, r_{12})\right|^2 = 8\pi^2 \int_0^{+\infty} dr_1\ r_1^2 \int_0^{+\infty} dr_2\ r_2^2 \int_{-1}^{+1} d\mu\ \left|\Psi^B_{h_1h_2}(r_1, r_2, r_{12})\right|^2 \tag{10}$$

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[2] We omit the antisymmetric color wave function and the plane wave for the center of mass motion which are common to all cases.
[3] A similar form lead to very good results in the case of baryons with a single heavy quark [12].

		AL1	AL2	AP1	AP2	BD
Ξ_{cc}	VAR	3612	3619	3629	3630	3639
	FAD [15]	3609	3616	3625	3628	3633
Ξ^*_{cc}	VAR	3706	3715	3722	3729	3722
Ξ_{bb}	VAR	10197	10180	10207	10179	10202
	FAD [15]	10194	10175	10204	10176	10197
Ξ^*_{bb}	VAR	10236	10219	10245	10219	10235
Ξ_{bc}	VAR	6919	6912	6933	6917	6936
	FAD [15]	6916	6913	6928	6907	6934
Ξ'_{bc}	VAR	6948	6942	6957	6944	6965
Ξ^*_{bc}	VAR	6986	6981	7000	6987	6993

		AL1	AL2	AP1	AP2	BD
Ω_{cc}	VAR	3702	3718	3711	3710	3743
	FAD [15]	3711	3718	3710	3709	3741
Ω^*_{cc}	VAR	3783	3802	3800	3802	3805
Ω_{bb}	VAR	10260	10249	10259	10226	10274
	FAD [15]	10267	10246	10258	10224	10271
Ω^*_{bb}	VAR	10297	10287	10301	10269	10302
Ω_{bc}	VAR	6986	6986	6990	6969	7013
	FAD [15]	7003	6996	6996	6971	7023
Ω'_{bc}	VAR	7009	7010	7011	6994	7033
Ω^*_{bc}	VAR	7046	7047	7055	7037	7057

TABLE II: Doubly heavy Ξ and Ω baryons masses in MeV. VAR stands for the results of our variational calculation. FAD stands for the results obtained in Ref. [15] using the same interquark interactions but within a Faddeev approach.

with μ being the cosine of the angle between the vectors $\vec{r}_1$ and $\vec{r}_2$ ($r_{12} = (\ r_1^2 + r_2^2 - 2r_1r_2\mu)^{1/2}$).

The functions ϕ_{h_1q} and ϕ_{h_2q} will be taken as the $S-$wave ground states $\varphi_j(r_j)$ of the single particle Hamiltonians H_j^{sp} of Eq. (4) modified at large distances

$$\phi_{h_jq}(r_j) \ = \ (1 + \alpha_j\, r_j)\,\varphi_j(r_j), \quad j = 1,2 \tag{11}$$

The heavy–heavy correlation function F^B will be given by a linear combination of gaussians[4]

$$F^B(r_{12}) \ = \ \sum_{j=1}^{4} a_j e^{-b_j^2(r_{12}+d_j)^2}, \quad a_1 = 1 \tag{12}$$

The value of one of the a_j parameters can be absorbed into the normalization constant N, so that we fix $a_1 = 1$. The values that we get for all parameters are given in appendix A

IV. STATIC PROPERTIES

A. Masses

The mass of the baryon is simply given by the expectation value of the intrinsic Hamiltonian. Our results (VAR) appear in Table II where we compare them with the ones obtained in Ref. [15] with the use of the same inter-quark interactions but within a Faddeev approach (FAD). For that purpose we have eliminated from the latter a small three-body force contribution of the type $V_{123} = \text{constant}/m_{h_1}m_{h_2}m_q$ that was also included in the evaluation of Ref. [15]. We will show their full results in the following tables. Whenever comparison is possible we find an excellent agreement between the two calculations. In some cases the variational masses are even lower than the Faddeev ones. Besides we give predictions for states that were not considered in the study of Ref. [15].

In Tables III and IV we compare our results with other theoretical calculations. Our central values correspond to the results obtained with the AL1 potential, while the errors quoted take into account the variation when using different potentials. The same presentation is used for the results obtained in Ref. [15] for which we now show their full values including the contribution of the three-body force. All calculations give similar results that vary within a few per cent. From the experimental point of view the SELEX Collaboration [1] has recently measured the value of $M_{\Xi_{cc}}$. This experimental value is 100 MeV smaller that our result. Note nevertheless that in Ref. [1] no account is given of

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[4] Note that F^B should vanish at large distances because of the confinement potential. The confinement potential is also responsible for the non–vanishing values of the parameters $\alpha_j,\ j = 1,2$ in Eq. (11).

	This work	[15]	[16]	[17]	[18]	[19]	[20]	[21]	[22]	[23]	[24]	[25]	Exp. [1]	Lattice [26]
Ξ_{cc}	3612^{+17}	3607^{+24}	3620	3480	3690	3740	3646	3524	3478	3660	3660	3610	3519 ± 1	3549 ± 95
Ξ^*_{cc}	3706^{+23}		3727	3610		3860	3733	3548	3610	3810	3740	3680		3641 ± 97
Ξ_{bb}	10197^{+10}_{-17}	10194^{+10}_{-19}	10202	10090	10160	10300			10093	10230	10340			
Ξ^*_{bb}	10236^{+9}_{-17}		10237	10130		10340			10133	10280	10370			
Ξ_{bc}	6919^{+17}_{-7}	6915^{+17}_{-9}	6933	6820	6960	7010			6820	6950	7040			
Ξ'_{bc}	6948^{+17}_{-6}		6963	6850		7070			6850	7000	6990			
Ξ^*_{bc}	6986^{+14}_{-5}		6980	6900		7100			6900	7020	7060			

TABLE III: Doubly heavy Ξ masses in MeV as obtained in different models. Our central values, and the ones of Ref. [15], have been evaluated with the AL1 potential. We also give the experimental value for $M_{\Xi_{cc}}$ measured by the SELEX Collaboration [1] (Notice this experimental mass does not show the systematic error), and the lattice results for $M_{\Xi_{cc}}$ and $M_{\Xi^*_{cc}}$ evaluated by the UKQCD Collaboration [26].

	This work	[15]	[16]	[17]	[18]	[19]	[20]	[22]	[23]	[24]	[25]	Lattice [26]
Ω_{cc}	3702^{+41}	3710^{+29}_{-2}	3778	3590	3860	3760	3749	3590	3760	3740	3710	3663 ± 97
Ω^*_{cc}	3783^{+22}		3872	3690		3900	3826	3690	3890	3820	3760	3734 ± 98
Ω_{bb}	10260^{+14}_{-34}	10267^{+4}_{-43}	10359	10180	10340	10340		10180	10320	10370		
Ω^*_{bb}	10297^{+5}_{-28}		10389	10200		10380		10200	10360	10400		
Ω_{bc}	6986^{+27}_{-17}	7003^{+20}_{-32}	7088	6910	7130	7050		6910	7050	7090		
Ω'_{bc}	7009^{+24}_{-15}		7116	6930		7110		6930	7090	7060		
Ω^*_{bc}	7046^{+11}_{-9}		7130	6990		7130		6990	7110	7120		

TABLE IV: Same as Table III for doubly heavy Ω baryons.

the systematic error. There are also lattice determinations by the UKQCD Collaboration [26] of the $M_{\Xi_{cc}}$, $M_{\Xi^*_{cc}}$ and $M_{\Omega_{cc}}$, $M_{\Omega^*_{cc}}$ masses. Our results, as most other theoretical values, are within errors of the lattice data.

The lattice calculation of Ref. [26] has also made an independent determination of the splittings which are given by

$$M_{\Xi^*_{cc}} - M_{\Xi_{cc}}\big|_{lattice} = 87 \pm 19\,\mathrm{MeV} \qquad\qquad M_{\Omega^*_{cc}} - M_{\Omega_{cc}}\big|_{lattice} = 67 \pm 16\,\mathrm{MeV} \tag{13}$$

Our results agree with the above splittings within errors.

B. Charge and mass densities and radii

The baryon charge density at the point P (coordinate vector $\vec{r}$ in the CM frame, see Fig. 1) is given by:

$$\begin{aligned} \rho^B_e(\vec{r}\,) &= \int d^3r_1 d^3r_2 \left|\Psi^B_{h_1h_2}(r_1, r_2, r_{12})\right|^2 \left\{e_{h_1}\delta^3(\vec{r} - \vec{y}_{h_1}) + e_{h_2}\delta^3(\vec{r} - \vec{y}_{h_2}) + e_q\delta^3(\vec{r} - \vec{y}_q)\right\} \\ &\equiv \rho^B_e(\vec{r}\,)\big|_{h_1} + \rho^B_e(\vec{r}\,)\big|_{h_2} + \rho^B_e(\vec{r}\,)\big|_q \end{aligned} \tag{14}$$

where $e_{h_1,\,h_2,\,q}$ are the quark charges in proton charge units e, and from Fig. 1 we have[5] $\vec{y}_{h_1} = \vec{y}_q + \vec{r}_1$, $\vec{y}_{h_2} = \vec{y}_q + \vec{r}_2$ and $\vec{y}_q = -\left(m_{h_1}\vec{r}_1 + m_{h_2}\vec{r}_2\right)/\overline{M}$. Since our $L = 0$ wave functions only depend on scalars (r_1, r_2 and r_{12}) the charge density is spherically symmetric ($\rho^B_e(\vec{r}\,) = \rho^B_e(|\vec{r}\,|)$).

The charge form factor is defined as usual

$$\mathcal{F}^B_e(\vec{q}\,) = \int d^3r\; e^{\mathrm{i}\vec{q}\cdot\vec{r}}\rho^B_e(r) \tag{15}$$

and it only depends on $|\vec{q}\,|$. Its value at $\vec{q} = \vec{0}$ gives the baryon charge in units of the proton charge.

The charge mean square radii are defined

$$\langle r^2\rangle^B_e = \int d^3r\; r^2\rho^B_e(r) = 4\pi\int_0^{+\infty} dr\; r^4\rho^B_e(r) \tag{16}$$

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[5] There exists the obvious restriction $m_{h_1}\vec{y}_{h_1} + m_{h_2}\vec{y}_{h_2} + m_q\vec{y}_q = \vec{0}$.

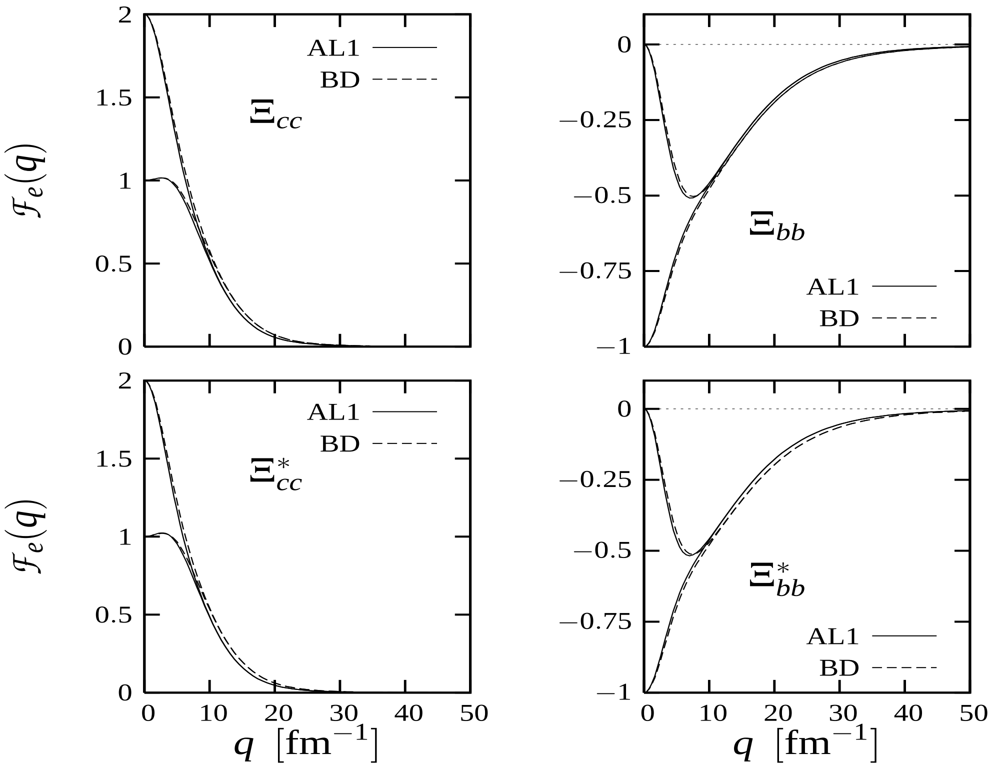

FIG. 2: Charge form factor of the Ξ_{cc}, Ξ_{bb} and Ξ^*_{cc}, Ξ^*_{bb} baryons evaluated with the AL1 [14, 15] (solid line) and BD [13] (dashed line) potentials. We show the two possible charge states.

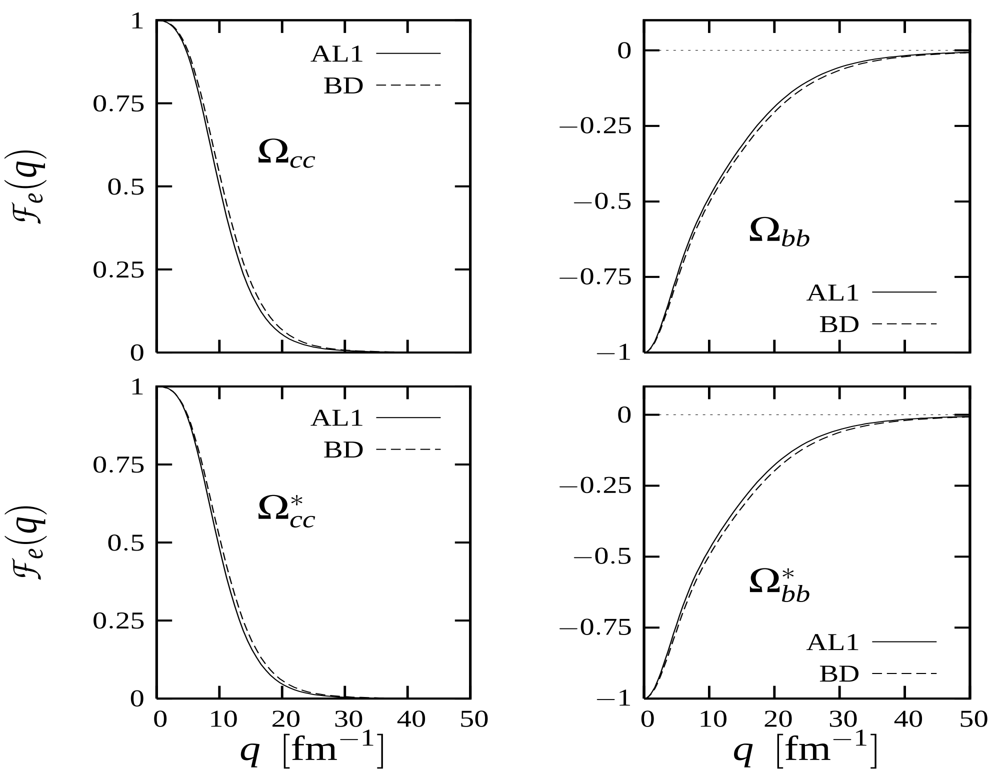

FIG. 3: Charge form factor of the Ω_{cc}, Ω_{bb} and Ω^*_{cc}, Ω^*_{bb} baryons evaluated with the AL1 (solid line) and BD (dashed line) potentials.

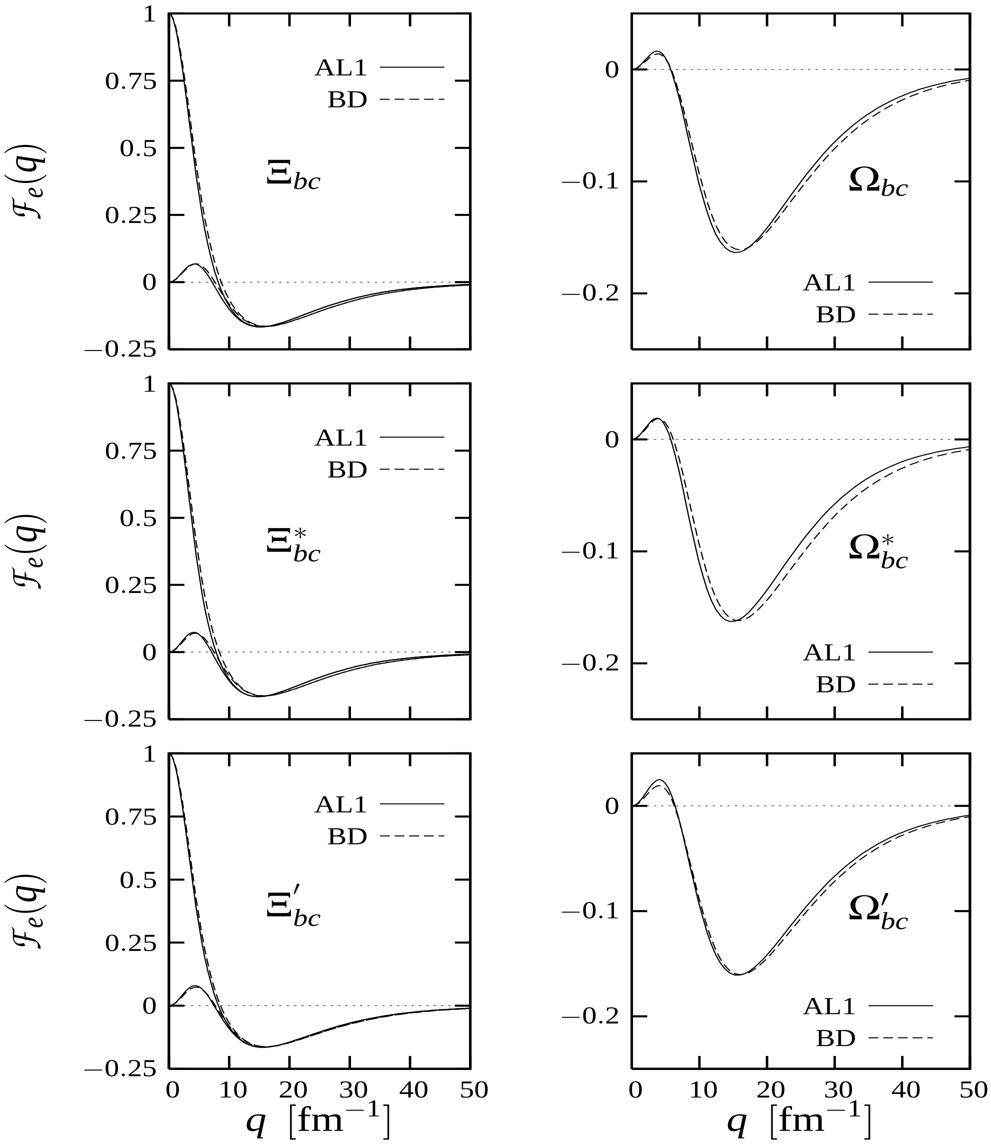

FIG. 4: Charge form factor of the Ξ_{bc}, Ξ^*_{bc}, Ξ'_{bc} and Ω_{bc}, Ω^*_{bc} Ω'_{bc} baryons evaluated with the AL1 (solid line) and BD (dashed line) potentials. For the Ξ baryons we show the two possible charge states.

In Figs. 2, 3 and 4 we show the charge form factors for the different doubly heavy baryons under study including the two different charge states for the doubly heavy Ξ ones. We show the calculations with both the AL1 potential of Refs. [14, 15] and the BD potential of Ref. [13]. The differences between the two calculations are minor in most cases.

In Table V we show the charge mean square radii. With the exceptions of the Ξ^0_{bc} and Ω^0_{bc}, we find good agreement with the results obtained in Ref. [15] within a Faddeev calculation. The possible presence of a $S_h = 0$ contribution in the wave functions of Ref. [15] could be the possible explanation for this discrepancy. We also compare with the results obtained, for a few states, in Ref. [28] with the use a relativistic quark model in the instant form. The agreement is bad in this case.

The baryon mass density, $\rho^B_m(r)$ is readily obtained from Eq. (14) with the obvious substitutions $(e_{h_1}, e_{h_2}, e_q) \rightarrow (m_{h_1}/\overline{M}, m_{h_2}/\overline{M}, m_q/\overline{M})$.

$$
\begin{aligned}
\rho^B_m(\vec{r}) &= \int d^3r_1 d^3r_2 \left|\Psi^B_{h_1h_2}(r_1, r_2, r_{12})\right|^2 \left\{ \frac{m_{h_1}}{\overline{M}} \delta^3(\vec{r} - \vec{y}_{h_1}) + \frac{m_{h_2}}{\overline{M}} \delta^3(\vec{r} - \vec{y}_{h_2}) + \frac{m_q}{\overline{M}} \delta^3(\vec{r} - \vec{y}_q) \right\} \\
&\equiv \rho^B_m(\vec{r})\big|_{h_1} + \rho^B_m(\vec{r})\big|_{h_2} + \rho^B_m(\vec{r})\big|_q
\end{aligned} \tag{17}
$$

	This work	[15]	[28]
Ξ_{cc}^{+}	$-0.030^{+0.003}_{-0.016}$	$-0.038^{+0.004}_{-0.016}$	0.1
Ξ_{cc}^{++}	$0.298^{+0.034}_{-0.028}$	$0.315^{+0.035}_{-0.030}$	1.3
Ξ_{cc}^{*+}	$-0.042^{+0.007}_{-0.019}$		
Ξ_{cc}^{*++}	$0.341^{+0.041}_{-0.042}$		
Ξ_{bb}^{0}	$0.221^{+0.033}_{-0.025}$	$0.242^{+0.035}_{-0.027}$	
Ξ_{bb}^{-}	$-0.133^{+0.014}_{-0.016}$	$-0.143^{+0.006}_{-0.018}$	
Ξ_{bb}^{*-}	$-0.142^{+0.018}_{-0.018}$		
Ξ_{bb}^{*0}	$0.238^{+0.035}_{-0.031}$		
Ξ_{bc}^{0}	$-0.057^{+0.006}_{-0.013}$	$-0.072^{+0.008}_{-0.017}$	
Ξ_{bc}^{+}	$0.279^{+0.026}_{-0.031}$	$0.306^{+0.035}_{-0.011}$	
$\Xi_{bc}^{\prime 0}$	$-0.065^{+0.010}_{-0.015}$		
$\Xi_{bc}^{\prime +}$	$0.283^{+0.036}_{-0.025}$		
Ξ_{bc}^{*0}	$-0.065^{+0.010}_{-0.018}$		
Ξ_{bc}^{*+}	$0.305^{+0.031}_{-0.039}$		

	This work	[15]	[28]
Ω_{cc}^{+}	$0.013^{+0.001}_{-0.002}$	$0.009_{-0.003}$	0.2
Ω_{cc}^{*+}	$0.009^{+0.001}_{-0.002}$		
Ω_{bb}^{-}	$-0.086^{+0.008}_{-0.001}$	$-0.090^{+0.007}_{-0.002}$	
Ω_{bb}^{*-}	$-0.092^{+0.011}_{-0.001}$		
Ω_{bc}^{0}	$-0.016_{+0.003}$	$-0.025^{+0.002}_{-0.003}$	
$\Omega_{bc}^{\prime 0}$	$-0.019^{+0.003}_{-0.003}$		
Ω_{bc}^{*0}	$-0.021^{+0.004}_{-0.002}$		

TABLE V: Charge mean square radii in fm^2 for doubly heavy Ξ and Ω baryons. Our central values, and the ones of Ref. [15], have been evaluated with the AL1 potential.

where we have normalized $\rho_m^B(r)$ to 1. Finally the mass mean square radii are defined

$$\langle r^2\rangle_m^B = \int d^3r\ r^2\rho_m^B(r) = 4\pi\int_0^{+\infty} dr\ r^4\rho_m^B(r) \tag{18}$$

In Figs. 5,6, 7 we show the mass densities for the different doubly heavy baryons under study. We show the mass densities, as defined in Eq. (17), for each quark flavor in the baryon and for the AL1 potential of Refs. [14, 15] and the BD potential of Ref. [13]. There are visible differences at small r mainly for light quarks. The differences at large r values are less significant as densities are very small in those cases.

The mass mean square radii are shown in Table VI. We find very good agreement with the results obtained in Ref. [15] within a Faddeev approach.

	This work	[15]
Ξ_{cc}	$0.081^{+0.002}_{-0.007}$	$0.083^{+0.002}_{-0.007}$
Ξ_{cc}^{*}	$0.089^{+0.002}_{-0.009}$	
Ξ_{bb}	$0.032_{-0.002}$	$0.033_{-0.002}$
Ξ_{bb}^{*}	$0.034_{-0.003}$	
Ξ_{bc}	$0.045_{-0.004}$	$0.046^{+0.001}_{-0.003}$
$\Xi_{bc}^{\prime}$	$0.044^{+0.001}_{-0.002}$	
Ξ_{bc}^{*}	$0.048_{-0.004}$	

	This work	[15]
Ω_{cc}	$0.078_{-0.006}$	$0.078_{-0.005}$
Ω_{cc}^{*}	$0.085^{+0.002}_{-0.008}$	
Ω_{bb}	$0.032_{-0.003}$	$0.032_{-0.002}$
Ω_{bb}^{*}	$0.034_{-0.004}$	
Ω_{bc}	$0.043_{-0.003}$	$0.045_{-0.003}$
$\Omega_{bc}^{\prime}$	$0.044_{-0.004}$	
Ω_{bc}^{*}	$0.047_{-0.005}$	

TABLE VI: Mass mean square radii in fm^2 for doubly heavy Ξ and Ω baryons. Our central values, and the ones of Ref. [15], have been evaluated with the AL1 potential.

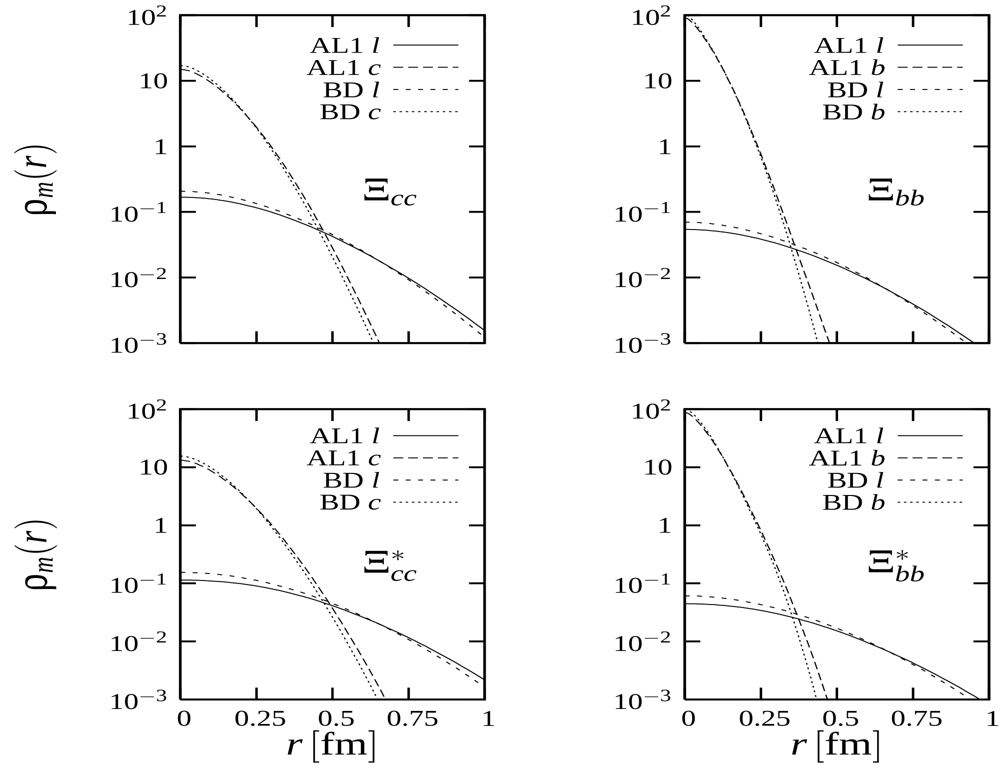

FIG. 5: Mass densities of the Ξ_{cc}, Ξ_{bb} and Ξ^*_{cc}, Ξ^*_{bb} baryons. Solid line: $l(u,d)$ quark mass density evaluated with the AL1 potential; long–dashed line: c or b quark mass density evaluated with the AL1 potential; short–dashed line: $l(u,d)$ quark mass density evaluated with the BD potential; dotted line: c or b quark mass density evaluated with the BD potential.

C. Magnetic moments

The orbital part of the magnetic moment is defined in terms of the velocities $\vec{v}$ of the quarks, with respect to the position of the CM, and it reads

$$\mu^B = \int d^3r_1 d^3r_2 \left(\Psi^B_{h_1h_2}(r_1,r_2,r_{12})\right)^* \left\{ \frac{e_{h_1}}{2m_{h_1}}(\vec{y}_{h_1}\times m_{h_1}\vec{v}_{h_1})_z + \frac{e_{h_2}}{2m_{h_2}}(\vec{y}_{h_2}\times m_{h_2}\vec{v}_{h_2})_z + \frac{e_q}{2m_q}(\vec{y}_q\times m_q\vec{v}_{y_q})_z \right\} \Psi^B_{h_1h_2}(r_1,r_2,r_{12}) \tag{19}$$

with[6] $m_{h_1}\vec{v}_{h_1} = -i\vec{\nabla}_1$, $m_{h_2}\vec{v}_{h_2} = -i\vec{\nabla}_2$ and $m_q\vec{v}_q = i\left(\vec{\nabla}_1 + \vec{\nabla}_2\right)$. Since our orbital wave function has $L=0$, the orbital magnetic moment vanishes. The magnetic moment of the baryon is then entirely given by the spin contribution.

$$\langle B;\, J,\, M_J = J|\; \frac{e_{h_1}}{2m_{h_1}}(\vec{\sigma}_{h_1})_z + \frac{e_{h_2}}{2m_{h_2}}(\vec{\sigma}_{h_2})_z + \frac{e_q}{2m_q}(\vec{\sigma}_q)_z\; |B;\, J,\, M_J = J\rangle \tag{20}$$

Those matrix elements are trivially evaluated with the results

$$\Xi_{h1h_2},\ \Omega_{h1h_2} \;\longrightarrow\; \frac{2}{3}\left(\frac{e_{h_1}}{2m_{h_1}} + \frac{e_{h_2}}{2m_{h_2}} - \frac{1}{2}\,\frac{e_q}{2m_q}\right)$$

[6] Note that the classical kinetic energy has a term on $\vec{v}_{h_1}\cdot\vec{v}_{h_2}$ and then the operator $m_{h_1}\vec{v}_{h_1}$ is not proportional to $-i\,\vec{\nabla}_{y_{h_1}}$, but it is rather given by $m_{h_1}\vec{v}_{h_1} = (\overline{M} - m_{h_1})/\overline{M}\cdot(-i\,\vec{\nabla}_{y_{h_1}}) - m_{h_2}/\overline{M}\cdot(-i\,\vec{\nabla}_{y_{h_2}}) = (-i\,\vec{\nabla}_1)$. Similarly $m_{h_2}\vec{v}_{h_2} = (-i\,\vec{\nabla}_2)$.

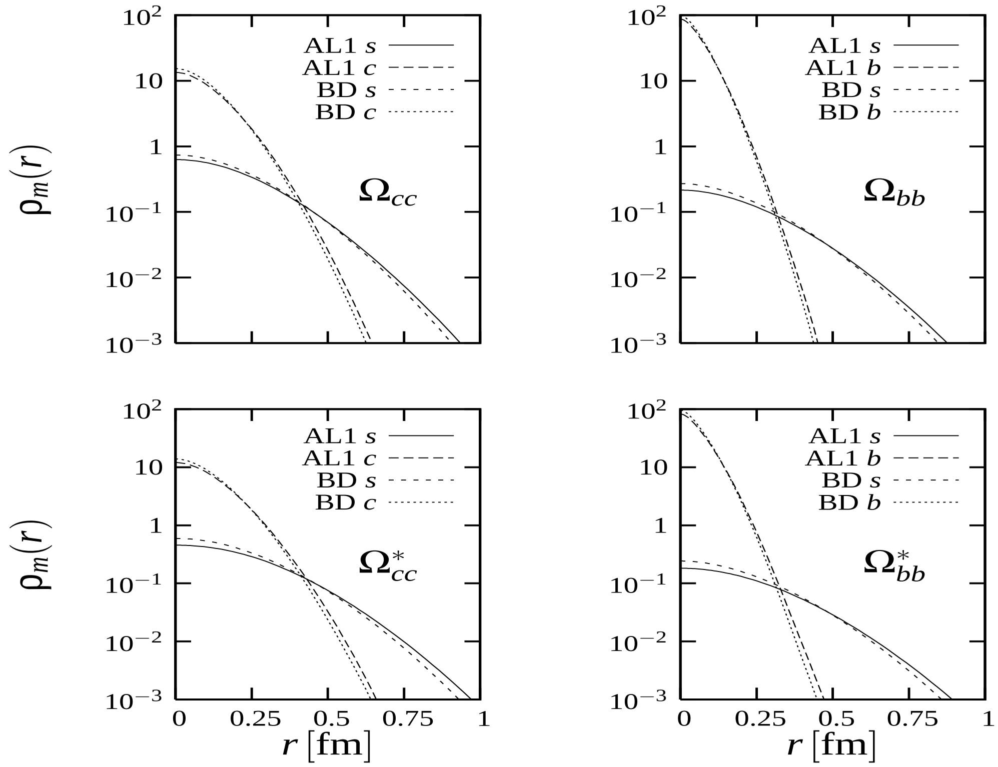

FIG. 6: Mass densities of the Ω_{cc}, Ω_{bb} and Ω^*_{cc}, Ω^*_{bb} baryons. Solid line: s quark mass density evaluated with the AL1 potential; long–dashed line: c or b quark mass density evaluated with the AL1 potential; short–dashed line: s quark mass density evaluated with the BD potential; dotted line: c or b quark mass density evaluated with the BD potential.

$$\begin{aligned} \Xi^*_{h1h_2},\ \Omega^*_{h1h_2} &\longrightarrow \frac{e_{h_1}}{2m_{h_1}} + \frac{e_{h_2}}{2m_{h_2}} + \frac{e_q}{2m_q} \\ \Xi'_{h1h_2},\ \Omega'_{h1h_2} &\longrightarrow \frac{e_q}{2m_q} \end{aligned} \tag{21}$$

In Table VII we give our numerical results. Our central values, as the ones obtained in Ref. [15] within a Faddeev approach, have been evaluated with the use of the AL1 potential. When compared to the values obtained in Ref. [15] we find very good agreement with just a few exceptions (Ξ^0_{bc}, Ξ^+_{bc}, Ω^0_{bc}). The discrepancy for the latter baryons may come from a possible non negligible $S_h = 0$ contribution to their wave functions in the calculation of Ref. [15]. In our case we have fixed $S_h = 1$ which we think is a very good approximation since in the limit of infinite heavy quark masses the spin of the heavy quark degrees of freedom is well defined. We also compare our results to the ones obtained in Refs.[2, 27, 28, 29, 30, 31] using different approaches[7]. The differences are large in some cases and sometimes even the signs are opposite. Being $L = 0$ a good approximation and with $m_b \gg m_u$, m_d, m_s, and to a lesser extent $m_c \gg m_u$, m_d, m_s, the values of the magnetic moments are essentially determined by the spin contribution of the light quark. To the extent that a fixed S_h is also a good approximation one would not expect to obtain different signs in different models.

V. SEMILEPTONIC DECAY

In this section we shall use the wave functions obtained with the variational method to study different doubly $B(1/2^+) \to B'(1/2^+)$ baryon semileptonic decays involving a $b \to c$ transition at the quark level.

[7] Note the definitions of Ξ_{bc} and Ξ'_{bc} are interchanged in Ref. [27], with Ξ_{bc} having $S_h = 0$ and Ξ'_{bc} having $S_h = 1$. The same applies to Ω_{bc} and Ω'_{bc}.

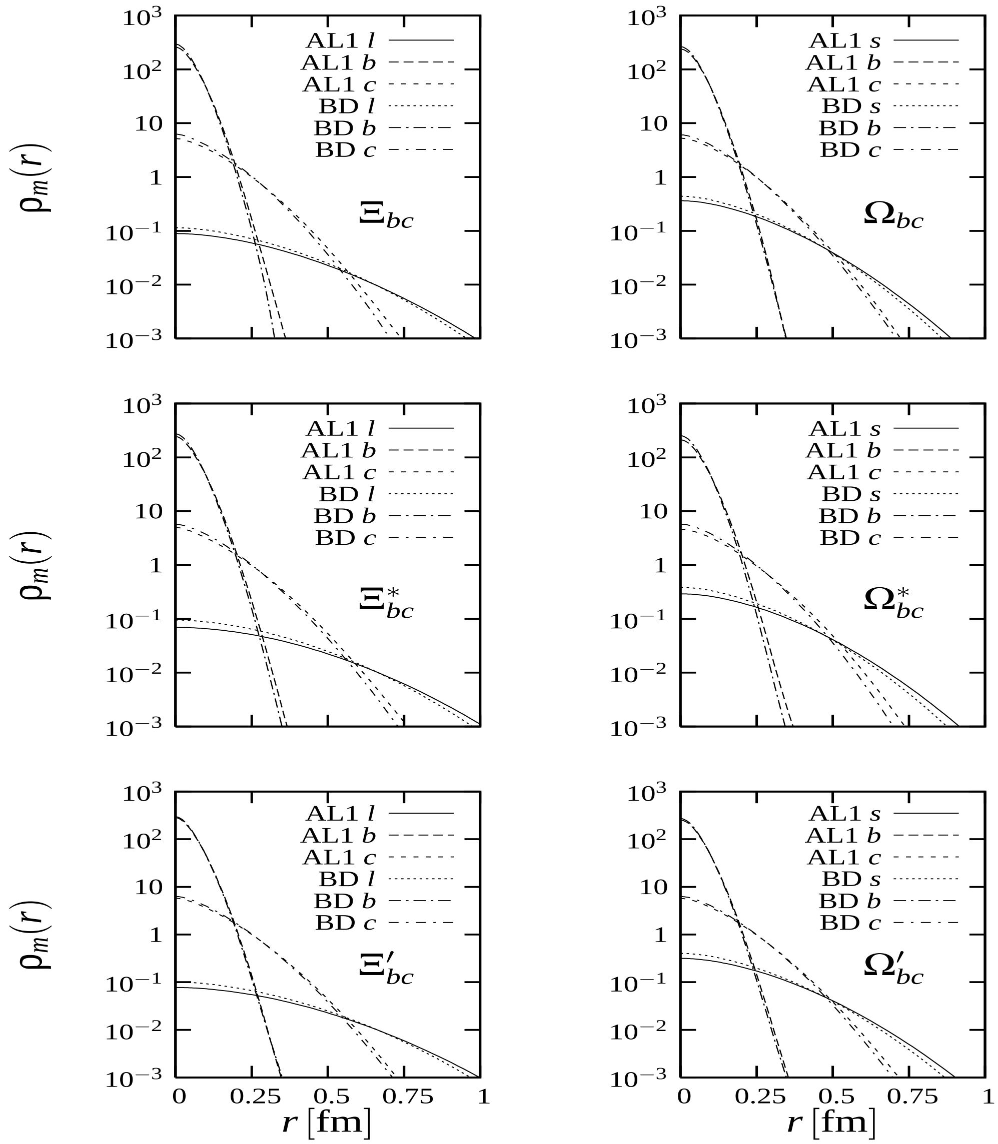

FIG. 7: Mass densities of the Ξ_{bc}, Ξ^*_{bc}, Ξ'_{bc} and Ω_{bc}, Ω^*_{bc}, Ω'_{bc} baryons. Solid line: $l(u,d)$ or s quark mass density evaluated with the AL1 potential; long–dashed line: b quark mass density evaluated with the AL1 potential, short–dashed line: c quark mass density evaluated with the AL1 potential; dotted line: $l(u,d)$ or s quark mass density evaluated with the BD potential; long-dashed dotted line: b quark mass density evaluated with the BD potential; short-dashed dotted line: c quark mass density evaluated with the BD potential.

The differential decay width reads

$$\mathrm{d}\Gamma = 8|V_{cb}|^2 m_{B'} G_F^2 \frac{d^3p'}{(2\pi)^3 2E'_{B'}} \frac{d^3k}{(2\pi)^3 2E_{\bar{\nu}_l}} \frac{d^3k'}{(2\pi)^3 2E'_l} (2\pi)^4 \delta^4(p-p'-k-k')\ \mathcal{L}^{\alpha\beta}(k,k')\mathcal{H}_{\alpha\beta}(p,p') \tag{22}$$

where $|V_{cb}|$ is the modulus of the corresponding Cabibbo–Kobayashi–Maskawa matrix element, $m_{B'}$ is the mass of the final baryon, $G_F = 1.1665 \times 10^{-11}\,\mathrm{MeV}^{-2}$[36] is the Fermi decay constant, p, p', k and k' are the four-momenta of the initial baryon, final baryon, final anti-neutrino and final lepton respectively, and $\mathcal{L}$ and $\mathcal{H}$ are the lepton and hadron tensors.

The lepton tensor is given as

$$\mathcal{L}^{\mu\sigma}(k,k') \;=\; k'^{\mu}k^{\sigma} + k'^{\sigma}k^{\mu} - g^{\mu\sigma} k\cdot k' + \mathrm{i}\epsilon^{\mu\sigma\alpha\beta} k'_{\alpha} k_{\beta} \tag{23}$$

where we use the convention $\epsilon^{0123} = -1$, $g^{\mu\mu} = (+,-,-,-)$.

	This work	[15]	[2]	[27]	[28]	[29]	[30]	[31]
Ξ_{cc}^{+}	$0.785^{+0.050}_{-0.030}$	$0.784^{+0.050}_{-0.029}$	0.806	0.72	0.72	$0.89 \sim 0.98$	$0.778 \sim 0.790$	0.86
Ξ_{cc}^{++}	$-0.208^{+0.035}_{-0.086}$	$-0.206^{+0.034}_{-0.086}$	-0.124	0.13	-0.10	-0.47	$-0.172 \sim -0.154$	0.17
Ξ_{cc}^{*+}	$-0.311^{+0.052}_{-0.130}$		-0.186			$-1.17 \sim -0.98$		0.20
Ξ_{cc}^{*++}	$2.67^{+0.27}_{-0.15}$		2.60			$3.16 \sim 3.18$		2.54
Ξ_{bb}^{0}	$-0.742^{+0.044}_{-0.091}$	$-0.742^{+0.044}_{-0.092}$		-0.53			$-0.726 \sim -0.705$	0.61
Ξ_{bb}^{-}	$0.251^{+0.045}_{-0.021}$	$0.251^{+0.046}_{-0.021}$		0.18			$0.226 \sim 0.236$	0.14
Ξ_{bb}^{*0}	$1.87^{+0.27}_{-0.13}$							1.37
Ξ_{bb}^{*-}	$-1.11^{+0.06}_{-0.14}$							-0.95
Ξ_{bc}^{0}	$0.518^{+0.048}_{-0.020}$	$0.058^{+0.059}_{-0.054}$		0.42				
Ξ_{bc}^{+}	$-0.475^{+0.040}_{-0.088}$	$-0.198^{+0.057}_{-0.056}$		-0.12				
$\Xi_{bc}^{\prime 0}$	$-0.993^{+0.065}_{-0.137}$			-0.76			$-0.385 \sim -0.366$	
$\Xi_{bc}^{\prime +}$	$1.99^{+0.27}_{-0.13}$			1.52			$1.50 \sim 1.54$	
Ξ_{bc}^{*0}	$-0.712^{+0.059}_{-0.133}$							-0.39
Ξ_{bc}^{*+}	$2.27^{+0.27}_{-0.14}$							2.04

	This work	[15]	[2]	[27]	[28]	[29]	[30]	[31]
Ω_{cc}^{+}	$0.635^{+0.012}_{-0.015}$	$0.635^{+0.011}_{-0.015}$	0.688	0.67	0.72	$0.59 \sim 0.64$	$0.657 \sim 0.663$	0.84
Ω_{cc}^{*+}	$0.139^{+0.009}_{-0.017}$		0.167			$-0.20 \sim 0.03$		0.39
Ω_{bb}^{-}	$0.101^{+0.007}_{-0.007}$	$0.101^{+0.007}_{-0.006}$		0.04			$0.105 \sim 0.108$	0.084
Ω_{bb}^{*-}	$-0.662^{+0.022}_{-0.024}$							-1.28
Ω_{bc}^{0}	$0.368^{+0.010}_{-0.011}$	$0.009^{+0.038}_{-0.029}$		0.45				
$\Omega_{bc}^{\prime 0}$	$-0.542^{+0.021}_{-0.024}$			-0.61			$-0.130 \sim -0.125$	
Ω_{bc}^{*0}	$-0.261^{+0.015}_{-0.021}$							-0.22

TABLE VII: Magnetic moments, in nuclear magnetons ($|e|/2m_p$, with m_p the proton mass), of doubly heavy Ξ and Ω baryons. Our central values, and the ones of Ref. [15], have been evaluated with the AL1 potential.

The hadron tensor is given as

$$\mathcal{H}_{\mu\sigma}(p,p') = \frac{1}{2}\sum_{r,r'}\left\langle B',r'\;\vec{p}\,'\left|\overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\right|B,r\;\vec{p}\right\rangle\;\left\langle B',r'\;\vec{p}\,'\left|\overline{\Psi}^c(0)\gamma_\sigma(I-\gamma_5)\Psi^b(0)\right|B,r\;\vec{p}\right\rangle^* \quad (24)$$

with $|B,r\;\vec{p}\rangle$ ($|B',r'\;\vec{p}\,'\rangle$) representing the initial (final) baryon with three–momentum $\vec{p}\,(\vec{p}\,')$ and spin third component r (r'). The baryon states are normalized such that $\langle r\;\vec{p}\,|\,r'\;\vec{p}\,'\rangle = (2\pi)^3(E(\vec{p}\,)/m)\,\delta_{rr'}\,\delta^3(\vec{p}-\vec{p}\,')$. The hadron matrix elements can be parametrized in terms of six form factors as

$$\left\langle B',r'\;\vec{p}\,'\left|\overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\right|B,r\;\vec{p}\right\rangle = \bar{u}^{B'}_{r'}(\vec{p}\,')\Big\{\gamma_\mu\left(F_1(w)-\gamma_5G_1(w)\right)+v_\mu\left(F_2(w)-\gamma_5G_2(w)\right) + v'_\mu\left(F_3(w)-\gamma_5G_3(w)\right)\Big\}u^B_r(\vec{p}\,) \quad (25)$$

where $u^{B,B'}$ are dimensionless Dirac spinors, normalized as $\bar{u}u=1$, and $v_\mu = p_\mu/m_B$ ($v'_\mu = p'_\mu/m_{B'}$) is the four velocity of the initial B (final B') baryon. The form factors are functions of the velocity transfer $w = v\cdot v'$ or equivalently of the four momentum transfer ($q = p-p'$) square $q^2 = m_B^2 + m_{B'}^2 - 2m_Bm_{B'}w$. In the decay w ranges from $w=1$, corresponding to zero recoil of the final baryon, to a maximum value given by $w = w_{\max} = (m_B^2+m_{B'}^2)/(2m_Bm_{B'})$ which depends on the transition.

Neglecting lepton masses, we have for the differential decay rates from transversely (Γ_T) and longitudinally (Γ_L) polarized W's (the total width is $\Gamma = \Gamma_L + \Gamma_T$) [37]

$$\begin{aligned}
\frac{\mathrm{d}\Gamma_T}{\mathrm{d}w} &= \frac{G_F^2|V_{cb}|^2}{12\pi^3} m_{B'}^3 \sqrt{w^2-1}\, q^2 \Big\{ (w-1)|F_1(w)|^2 + (w+1)|G_1(w)|^2 \Big\} \\
\frac{\mathrm{d}\Gamma_L}{\mathrm{d}w} &= \frac{G_F^2|V_{cb}|^2}{24\pi^3} m_{B'}^3 \sqrt{w^2-1} \Big\{ (w-1)|\mathcal{F}^V(w)|^2 + (w+1)|\mathcal{F}^A(w)|^2 \Big\} \\
\mathcal{F}^{V,A}(w) &= \Big[(m_B \pm m_{B'}) F_1^{V,A}(w) + (1 \pm w) \Big(m_{B'} F_2^{V,A}(w) + m_B F_3^{V,A}(w) \Big) \Big], \\
& \quad F_j^V \equiv F_j(w),\ F_j^A \equiv G_j(w),\ j = 1,2,3
\end{aligned} \tag{26}$$

One can also evaluate the polar angle distribution [37]:

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}w\,\mathrm{d}\cos\theta} = \frac{3}{8} \left(\frac{\mathrm{d}\Gamma_T}{\mathrm{d}w} + 2\frac{\mathrm{d}\Gamma_L}{\mathrm{d}w} \right) \Big\{ 1 + 2\alpha' \cos\theta + \alpha'' \cos^2\theta \Big\} \tag{27}$$

where θ is the angle between $\vec{k}'$ and $\vec{p}'$ measured in the off–shell W rest frame, and α' and α'' are asymmetry parameters given by

$$\begin{aligned}
\alpha' &= -\frac{G_F^2|V_{cb}|^2}{6\pi^3} m_{B'}^3 \frac{q^2\,(w^2-1)\,F_1(w)G_1(w)}{\mathrm{d}\Gamma_T/\mathrm{d}w + 2\,\mathrm{d}\Gamma_L/\mathrm{d}w} \\
\alpha'' &= \frac{\mathrm{d}\Gamma_T/\mathrm{d}w - 2\,\mathrm{d}\Gamma_L/\mathrm{d}w}{\mathrm{d}\Gamma_T/\mathrm{d}w + 2\,\mathrm{d}\Gamma_L/\mathrm{d}w}
\end{aligned} \tag{28}$$

These asymmetry parameters are functions of the velocity transfer w and on averaging over w the numerators and denominators are integrated separately and thus we have

$$\langle \alpha' \rangle = -\frac{G_F^2|V_{cb}|^2}{6\pi^3} \frac{m_{B'}^3}{\Gamma_T} \frac{\int_1^{w_{\max}} q^2\,(w^2-1)\,F_1(w)G_1(w)\mathrm{d}w}{1+2R_{L/T}}, \qquad \langle \alpha'' \rangle = \frac{1-2R_{L/T}}{1+2R_{L/T}}, \qquad R_{L/T} = \frac{\Gamma_L}{\Gamma_T} \tag{29}$$

A. Form factors

To obtain the form factors we have to evaluate the matrix elements

$$\Big\langle B', r'\ \vec{p}' \Big| \overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0) \Big| B, r\ \vec{p} \Big\rangle \tag{30}$$

which in our model are given by

$$\sqrt{\frac{E_B(\vec{p})}{m_B}} \sqrt{\frac{E_{B'}(\vec{p}')}{m_{B'}}}\ {}_{NR}\Big\langle B', r'\ \vec{p}' \Big| \overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0) \Big| B, r\ \vec{p} \Big\rangle_{NR} \tag{31}$$

where the suffix "NR" denotes our nonrelativistic states and the factors $\sqrt{E/m}$ take into account the different normalization. We shall work in the initial baryon rest frame so that $\vec{p} = \vec{0}$, $\vec{p}' = -\vec{q}$, and take $\vec{q}$ in the positive z direction. Furthermore we shall use the spectator approximation. Having all this in mind we have in momentum space

$$\begin{aligned}
& \sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\ {}_{NR}\Big\langle B', r'\ -\vec{q} \Big| \overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0) \Big| B, r\ \vec{0} \Big\rangle_{NR} \\
&\quad = 2\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}} \sum_{s_1}\sum_{s_2} \left(\frac{1}{2}\frac{1}{2} S_h \Big| s_1, s_2-s_1, s_2 \right) \left(S_h \frac{1}{2}\frac{1}{2} \Big| s_2, r-s_2, r \right) \\
&\qquad \times \left(\frac{1}{2}\frac{1}{2} S'_h \Big| r'-r+s_1, s_2-s_1, r'-r+s_2 \right) \left(S'_h \frac{1}{2}\frac{1}{2} \Big| r'-r+s_2, r-s_2, r' \right)
\end{aligned}$$

$$\times \int d^3q_1\, d^3q_2 \left(\Phi^{B'}_{c\,h_2}(\vec{q}_1 - \frac{m_{h_2}+m_q}{\overline{M}\,'}\,\vec{q},\, \vec{q}_2 + \frac{m_{h_2}}{\overline{M}\,'}\,\vec{q}) \right)^* \Phi^{B}_{b\,h_2}(\vec{q}_1,\vec{q}_2)$$
$$\times \sqrt{\frac{m_b}{E_b(\vec{q}_1)}} \sqrt{\frac{m_c}{E_c(\vec{q}_1-\vec{q})}}\; \bar{u}^c_{r'-r+s_1}(\vec{q}_1-\vec{q})\gamma_\mu(I-\gamma_5)\, u^b_{s_1}(\vec{q}_1) \tag{32}$$

where $\Phi^{B}_{b\,h_2}(\vec{q}_1,\vec{q}_2)$ ($\Phi^{B'}_{c\,h_2}(\vec{q}_1,\vec{q}_2)$) is the Fourier transform of the coordinate space wave function $\Psi^{B}_{b\,h_2}(r_1,r_2,r_{12})$ ($\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})$) with $\vec{q}_1$, $\vec{q}_2$ being the conjugate momenta to the space variables $\vec{r}_1$, $\vec{r}_2$. The factor of two comes from the fact that: i) for $bc-$baryon decays, the charm quark resulting from the $b \to c$ transition could be either the particle 1 or the particle 2 in the final cc baryon, while ii) for $bb-$baryon decays, there exist two equal contributions resulting for the decay of each of the two bottom quarks of the initial baryon.

The actual calculation is done in coordinate space where we have

$$\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\;{}_{NR}\left\langle B',r' \;\; -\vec{q} \right| \overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0) \left| B, r\;\vec{0} \right\rangle_{NR}$$
$$= 2\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}} \sum_{s_1}\sum_{s_2} \left(\frac{1}{2}\frac{1}{2}S_h \middle| s_1, s_2-s_1, s_2\right) \left(S_h\frac{1}{2}\frac{1}{2}\middle| s_2, r-s_2, r\right)$$
$$\times \left(\frac{1}{2}\frac{1}{2}S'_h \middle| r'-r+s_1, s_2-s_1, r'-r+s_2\right)\left(S'_h\frac{1}{2}\frac{1}{2}\middle| r'-r+s_2, r-s_2, r'\right)$$
$$\times \int d^3r_1\, d^3r_2\, \Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})\; e^{i\frac{m_{h_2}}{\overline{M}\,'}\,\vec{q}\cdot\vec{r}_2}\; e^{-i\frac{m_{h_2}+m_q}{\overline{M}\,'}\,\vec{q}\cdot\vec{r}_1}$$
$$\times \sqrt{\frac{m_b}{E_b(\vec{l}\,)}} \sqrt{\frac{m_c}{E_c(\vec{l}-\vec{q})}}\; \bar{u}^c_{r'-r+s_1}(\vec{l}-\vec{q})\gamma_\mu(I-\gamma_5)\, u^b_{s_1}(\vec{l}\,)\;\Psi^{B}_{b\,h_2}(r_1,r_2,r_{12}) \tag{33}$$

where $\vec{l} = -i\,\vec{\nabla}_1$ represents an internal momentum which is much smaller than the heavy quark masses m_b, m_c. On the other hand $|\vec{q}|$ can be large[8]. Thus, to evaluate the above expression we have made use of an expansion in $\vec{l}$, introduced in Ref. [38], where second order terms in $\vec{l}$ are neglected, while all orders in $|\vec{q}|$ are kept. For instance $E_c(\vec{l}-\vec{q})$ is approximated by $E_c(\vec{l}-\vec{q}) \approx E_c(\vec{q}) \times (1 - \vec{l}\cdot\vec{q}/E_c^2(\vec{q}))$ with $E_c(\vec{q}) = \sqrt{m_c^2+\vec{q}^{\,2}}$.

The three vector and three axial form factors can be extracted from the set of equations[9]

$$\begin{aligned}
\left\langle B', 1/2 \;\; -\vec{q}\right| \overline{\Psi}^c(0)\gamma_1\Psi^b(0)\left| B, -1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\;\frac{|\vec{q}\,|}{E_{B'}(-\vec{q})+m_{B'}}\;F_1(|\vec{q}\,|)\\
\left\langle B', 1/2 \;\; -\vec{q}\right| \overline{\Psi}^c(0)\gamma_3\Psi^b(0)\left| B, 1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\;|\vec{q}\,|\left(\frac{F_1(|\vec{q}\,|)}{E_{B'}(-\vec{q})+m_{B'}} + \frac{F_3(|\vec{q}\,|)}{m_{B'}}\right)\\
\left\langle B', 1/2 \;\; -\vec{q}\right| \overline{\Psi}^c(0)\gamma_0\Psi^b(0)\left| B, 1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\left(F_1(|\vec{q}\,|)+F_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q})}{m_{B'}}\,F_3(|\vec{q}\,|)\right)
\end{aligned} \tag{34}$$

for the vector form factors and

$$\begin{aligned}
\left\langle B', 1/2 \;\; -\vec{q}\right| \overline{\Psi}^c(0)\gamma_1\gamma_5\Psi^b(0)\left| B, -1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\;(-G_1(|\vec{q}\,|))\\
\left\langle B', 1/2 \;\; -\vec{q}\right| \overline{\Psi}^c(0)\gamma_3\gamma_5\Psi^b(0)\left| B, 1/2\;\vec{0}\right\rangle &= \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}}\;\left(-G_1(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,G_3(|\vec{q}\,|)}{m_{B'}(E_{B'}(-\vec{q})+m_{B'})}\right)
\end{aligned}$$

---

[8] At $q^2 = 0$ one has $|\vec{q}\,| = (m_B^2 - m_{B'}^2)/2m_B$ which is $\approx m_B/3$ for the transitions under study.
[9] Remember $\vec{q}$ is in the z direction. Notice also that, for $\vec{p} = \vec{0}$, w is just a function of $|\vec{q}\,|$.

$$\left\langle B', 1/2 \; -\vec{q} \left| \overline{\Psi}^c(0)\gamma_0\gamma_5\Psi^b(0) \right| B, 1/2 \; \vec{0} \right\rangle = \sqrt{\frac{E_{B'}(-\vec{q})+m_{B'}}{2m_{B'}}} \frac{|\vec{q}\,|}{E_{B'}(-\vec{q})+m_{B'}} \Bigg(-G_1(|\vec{q}\,|)+G_2(|\vec{q}\,|) + \frac{E_{B'}(-\vec{q})}{m_{B'}} G_3(|\vec{q}\,|) \Bigg) \tag{35}$$

for the axial ones. All the left hand side terms can be evaluated using Eq.(33) with the approximation mentioned above.

For each transition there are only two different coordinate space integrals from which all different matrix elements can be evaluated. Those integrals are

$$\begin{aligned} \mathcal{I}^{B'B}(|\vec{q}\,|) &= \int d^3r_1\, d^3r_2\, e^{i\frac{m_{h_2}}{\overline{M}'}\vec{q}\cdot\vec{r}_2}\, e^{-i\frac{m_{h_2}+m_q}{\overline{M}'}\vec{q}\cdot\vec{r}_1} \left[\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})\right]^* \Psi^{B}_{b\,h_2}(r_1,r_2,r_{12}) \\ \mathcal{K}^{B'B}(|\vec{q}\,|) &= \frac{1}{|\vec{q}\,|^2}\int d^3r_1\, d^3r_2\, e^{i\frac{m_{h_2}}{\overline{M}'}\vec{q}\cdot\vec{r}_2}\, e^{-i\frac{m_{h_2}+m_q}{\overline{M}'}\vec{q}\cdot\vec{r}_1} \left[\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})\right]^* \vec{l}\cdot\vec{q}\; \Psi^{B}_{b\,h_2}(r_1,r_2,r_{12}) \end{aligned} \tag{36}$$

In appendix B we relate the form factors to the integrals $\mathcal{I}^{B'B}(|\vec{q}\,|)$ and $\mathcal{K}^{B'B}(|\vec{q}\,|)$ for the different S_h, S'_h combinations, while in appendix C we give the actual expressions we use to evaluate those integrals.

1. *Current conservation*

In the limit $m_b = m_c$ and for $B' = B$ (and thus $S_h = S'_h$) vector current conservation provides a relation among the vector F_2 and F_3 form factors, namely

$$F_2(w) = F_3(w) \tag{37}$$

On the other hand the matrix element of the zeroth component of the vector current evaluated at $w = 1$ just counts the number of heavy quarks so that we should have

$$F_1(1) + F_2(1) + F_3(1) = 2 \tag{38}$$

In this limiting situation the integrals $\mathcal{I}^{BB}(|\vec{q}\,|)$ and $\mathcal{K}^{BB}(|\vec{q}\,|)$ are related by[10]

$$\mathcal{K}^{BB}(|\vec{q}\,|) = \frac{m_{h_2}+m_q}{2\overline{M}}\, \mathcal{I}^{BB}(|\vec{q}\,|) \tag{39}$$

Besides one has that $\mathcal{I}^{BB}(0) = 1$.

Using now the relations in Eq.(B4) in appendix B we see that our model satisfies the constraint in Eq.(38) exactly. On the other hand we violate current conservation . For instance, and again using the relations in Eq.(B4), we obtain for $w = 1$

$$F_2(1) = F_3(1) + 2(1 - \frac{m_B}{\overline{M}}) \tag{40}$$

which shows that current conservation is violated by a term proportional to the binding energy of the baryon divided by the sum of the masses of its constituents. Improvements on vector current conservation would require at minimum the introduction of two–body currents [39], going thus beyond the spectator approximation, that we have not considered in this analysis. This deficiency is shared by all previous calculations [32, 33, 34, 35].

Note also that, for transitions that do not conserve the spin of the heavy quark subsystem S_h (i.e. $\Xi'^0_{bc} \to \Xi^+_{cc} l\bar{\nu}_l$) we have in the $m_b = m_c$ limit and at zero reocoil that

$$F_1(1) + F_2(1) + F_3(1) = 0 \tag{41}$$

due to the orthogonality of the initial and final baryon wave–functions.

[10] One just has to integrate by parts in the $\mathcal{K}^{BB}(|\vec{q}\,|)$ expression

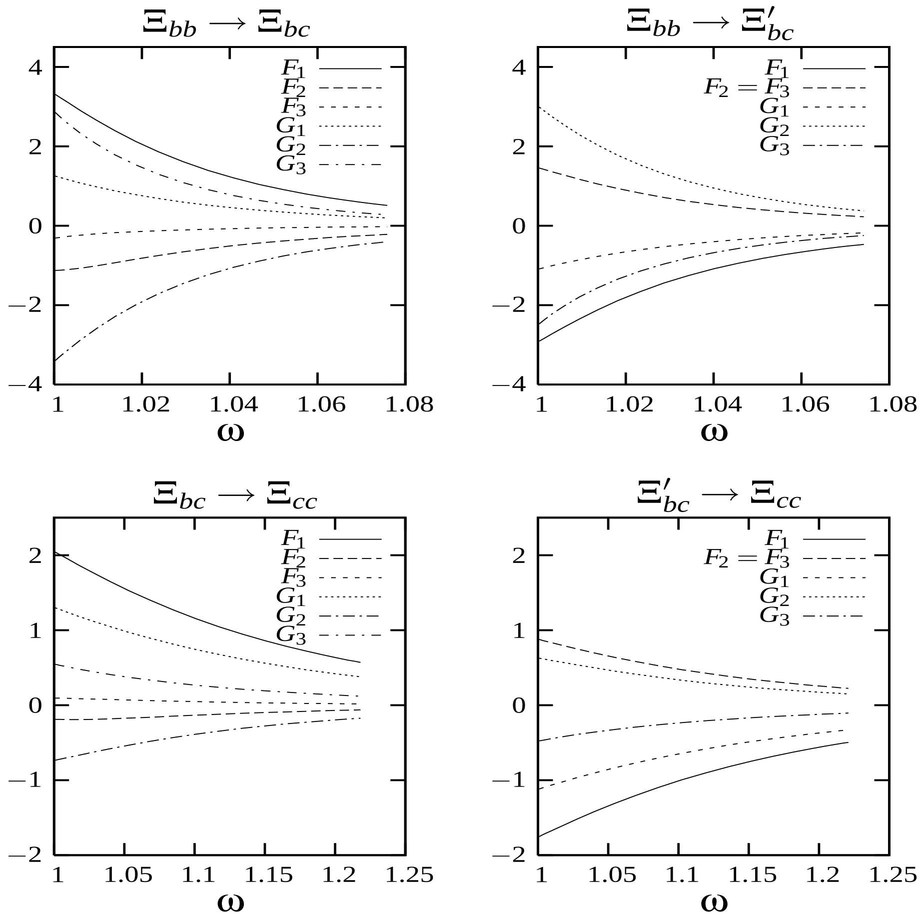

FIG. 8: Vector F_1, F_2, F_3 and axial G_1, G_2, G_3 form factors for doubly $\Xi(J = 1/2)$ baryons decays evaluated with the AL1 potential.

B. Results

In Figs. 8, 9 we show the form factors for the different transitions evaluated with the AL1 potential. Variations when using a different potential are at the level a few per cent at most. The results for doubly heavy Ξ decays are almost identical to the corresponding ones for doubly heavy Ω decays. The fact that we have two heavy quarks and that the light one acts as a spectator makes the results almost independent of the light quark mass.

In Figs. 10, 11 we show now our results for the differential $d\Gamma_T/dw$, $d\Gamma_L/dw$ and $d\Gamma/dw$ decay widths evaluated with the AL1 and BD potentials. The differences between the results obtained with the two inter-quark interactions could reach 30% for some transitions and for some regions of w. As a consequence of the apparent SU(3) symmetry in the form factors we also find that the results for doubly heavy Ξ and Ω decays are very close to each other. This apparent SU(3) symmetry goes over to the integrated decay widths and asymmetry parameters.

In Table VIII we give our results for the semileptonic decay width (transverse Γ_T, longitudinal Γ_L and total Γ) for the different processes under study. Our central values have been evaluated using the AL1 potential while the errors show the variations when changing the interaction. The biggest variations appear for the BD potential for which one obtains results which are larger by $7 \sim 12\%$. In Table IX we compare our results to the ones calculated in different models. For that purpose we need a value for $|V_{cb}|$ for which we take $|V_{cb}| = 0.0413$. Our results are in reasonable agreement with the ones in Ref. [32] where they use a relativistic quark model evaluated in the quark-diquark approximation, and with the $\Gamma(\Xi_{bc} \to \Xi_{cc})$ value of Ref. [35] obtained using HQET. The value for the latter width but now evaluated in the relativistic three–quark model calculation of Ref. [33] is much smaller than in any other calculation. On the other hand in Ref. [34], where they use the Bethe–Salpeter equation applied to a quark-diquark system, they obtain much larger results for all transitions.

In Table X we compile our results for the average angular asymmetries α' and α'', as well as the $R_{L/T}$ ratio, introduced in Eq.(29). The central values have been obtained with the AL1 potential. Being all quantities ratios the

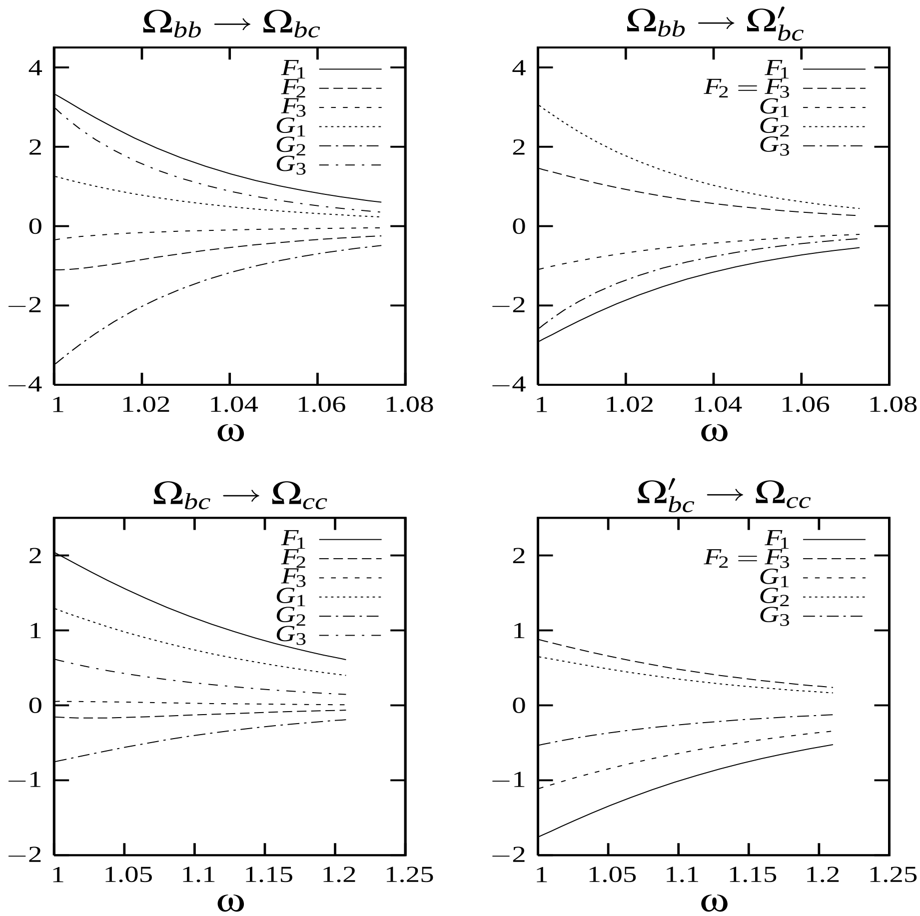

FIG. 9: Same as Fig. 8 for doubly heavy $\Omega(J = 1/2)$ baryons decays.

	Γ_T	Γ_L	Γ
$\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l$	$0.97^{+0.10}_{-0.02}$	$1.28^{+0.19}_{-0.04}$	$2.25^{+0.29}_{-0.06}$
$\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l$	$1.15^{+0.08}_{-0.01}$	$1.86^{+0.}_{-0.02}$	$3.01^{+0.30}_{-0.03}$
$\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l$	$0.73^{+0.08}_{-0.02}$	$0.52^{+0.07}_{-0.01}$	$1.24^{+0.15}_{-0.03}$
$\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l$	$0.89^{+0.05}_{-0.02}$	$0.70^{+0.06}_{-0.01}$	$1.59^{+0.11}_{-0.03}$

	Γ_T	Γ_L	Γ
$\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l$	$1.06^{+0.07}_{-0.01}$	$1.45^{+0.16}_{-0.01}$	$2.51^{+0.23}_{-0.02}$
$\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l$	$1.15^{+0.06}$	$1.88^{+0.17}$	$3.03^{+0.23}$
$\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l$	$0.79^{+0.08}$	$0.57^{+0.07}$	$1.36^{+0.15}$
$\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l$	$0.89^{+0.05}$	$0.70^{+0.05}$	$1.59^{+0.10}$

TABLE VIII: Semileptonic decay widths in units of $|V_{cb}|^2 \cdot 10^{-11}$ GeV. Γ_T and Γ_L stand for the transverse and longitudinal contributions to the width Γ. The central values have been obtained with the AL1 potential. l stands for a light charged lepton, $l = e,\, \mu$

	This work	[32]	[33]	[34]	[35]
$\Gamma(\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l)$	$3.84^{+0.49}_{-0.10}$	3.26		28.5	
$\Gamma(\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$5.13^{+0.51}_{-0.05}$	4.59	0.79	8.93	4.0
$\Gamma(\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l)$	$2.12^{+0.26}_{-0.05}$	1.64		4.28	
$\Gamma(\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$2.71^{+0.19}_{-0.05}$	1.76		7.76	

	This work	[32]	[34]
$\Gamma(\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l)$	$4.28^{+0.39}_{-0.03}$	3.40	28.8
$\Gamma(\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$5.17^{+0.39}$	4.95	
$\Gamma(\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l)$	$2.32^{+0.26}$	1.66	
$\Gamma(\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$2.71^{+0.17}$	1.90	

TABLE IX: Semileptonic decay widths in units of 10^{-14} GeV. We have used a value $|V_{cb}| = 0.0413$. l stands for a light charged lepton, $l = e,\, \mu$

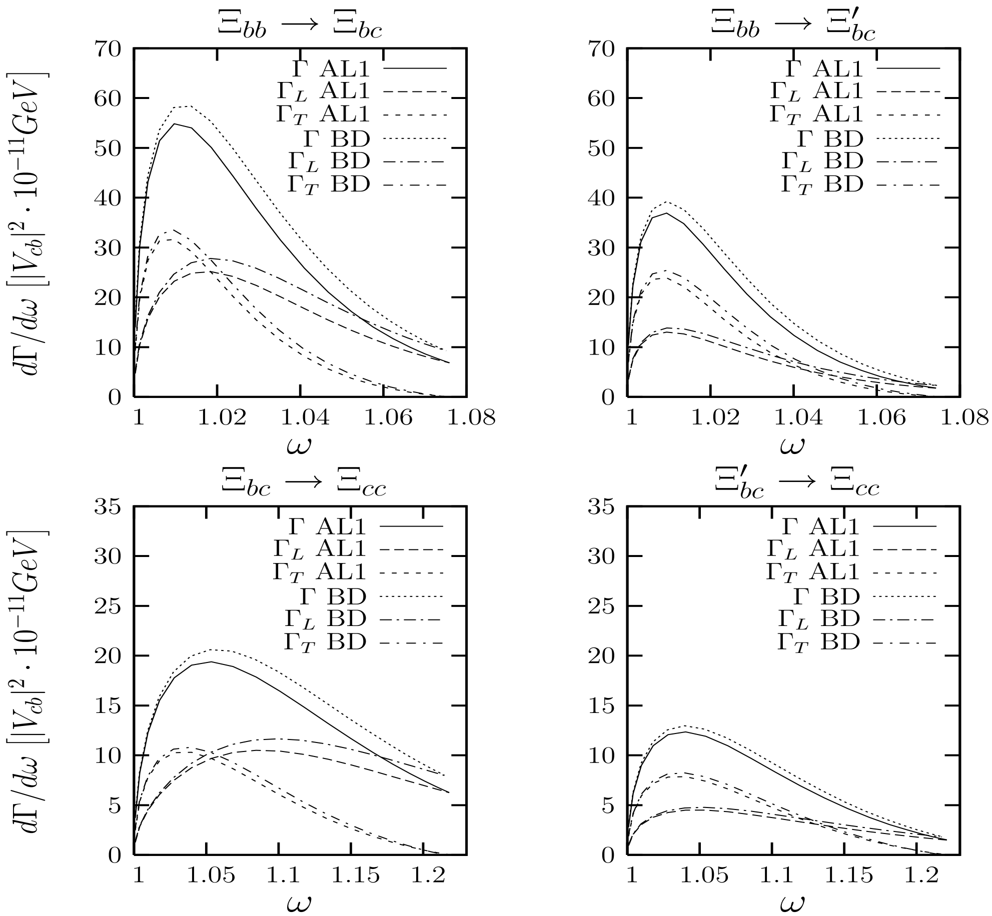

FIG. 10: $d\Gamma/dw$, $d\Gamma_L/dw$ and $d\Gamma_T/dw$ semileptonic decay widths in units of $|V_{cb}|^2 \cdot 10^{-11}$ GeV, for doubly $\Xi(J = 1/2)$ baryons decays. Solid line, long–dashed line and short–dashed line: $d\Gamma/dw$, $d\Gamma_L/dw$ and $d\Gamma_T/dw$ evaluated with the AL1 potential; dotted line, long–dashed dotted line and short–dashed dotted line: $d\Gamma/dw$, $d\Gamma_L/dw$ and $d\Gamma_T/dw$ evaluated with the BD potential.

	$\langle\alpha'\rangle$	$\langle\alpha''\rangle$	$R_{L/T}$
$\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l$	$-0.13^{+0.01}$	$-0.45_{-0.02}$	$1.33^{+0.06}_{-0.01}$
$\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l$	$-0.12^{+0.01}$	$-0.53_{-0.01}$	$1.62^{+0.07}$
$\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l$	-0.19	$-0.17_{-0.01}$	$0.71^{+0.01}_{-0.01}$
$\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l$	-0.19	$-0.23_{-0.01}$	$0.79^{+0.02}$

	$\langle\alpha'\rangle$	$\langle\alpha''\rangle$	$R_{L/T}$
$\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l$	$-0.13^{+0.01}$	$-0.47_{-0.01}$	$1.37^{+0.06}$
$\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l$	$-0.12^{+0.01}$	$-0.53_{-0.01}$	$1.63^{+0.06}$
$\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l$	$-0.19_{-0.01}$	$-0.18_{-0.01}$	$0.72^{+0.02}$
$\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l$	-0.19	$-0.23_{-0.01}$	$0.79^{+0.02}$

TABLE X: Averaged values of the asymmetry parameters α' and α'' evaluated as indicated in Eq.(29). We also show the ratio $R_{L/T} = \Gamma_L/\Gamma_T$. The central values have been obtained with the AL1 potential. l stands for a light charged lepton, $l = e$, μ

variation when changing the inter-quark interaction are in most cases small.

VI. SUMMARY

We have evaluated static properties and semileptonic decays for the ground state of doubly heavy Ξ and Ω baryons. The calculations have been done in the framework of a nonrelativistic quark model with the use of five different inter-quark interactions. The use of different quark-quark potentials allows us to obtain an estimation of the theoretical

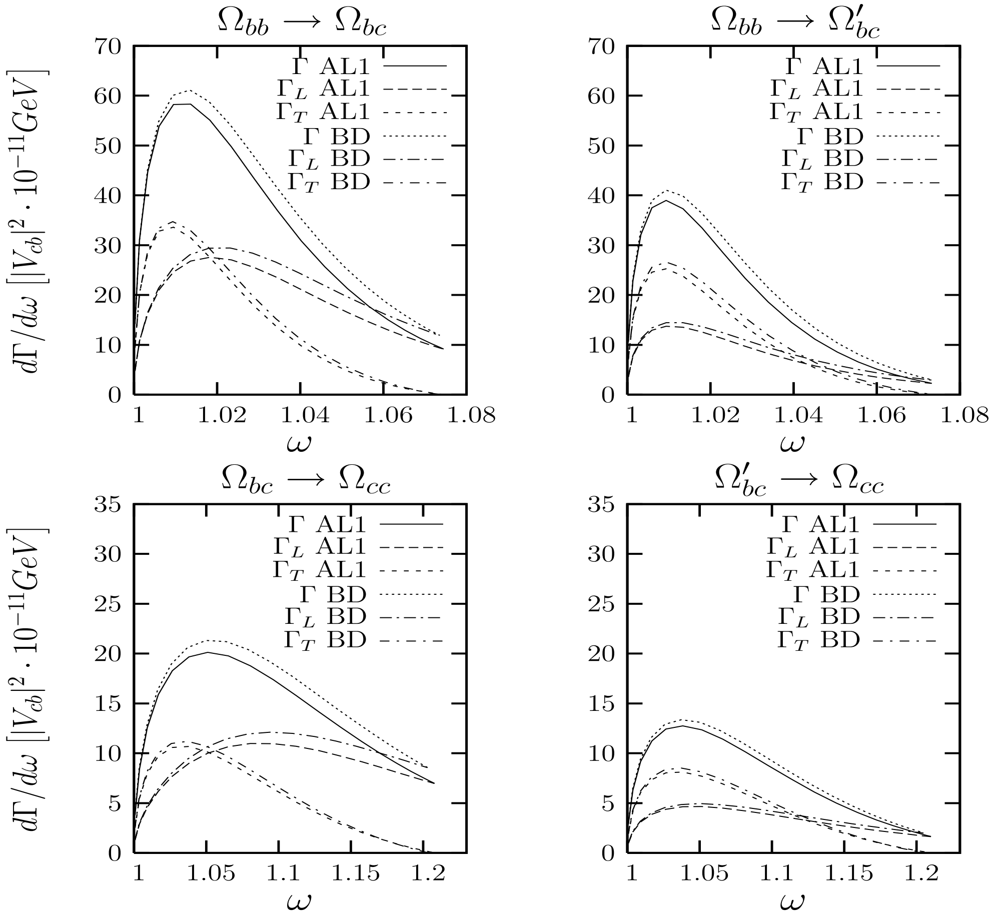

FIG. 11: Same as Fig. 10 for doubly heavy $\Omega(J = 1/2)$ baryons decays.

uncertainties. In order to build our wave functions we have made use of the constraints imposed by the infinite heavy quark mass limit. In this limit the spin–spin interactions vanish and the total spin of the two heavy quarks is well defined. With this approximation we have used a simple variational approach, with Jastrow type orbital wave functions, to solve the involved three-body problem.

Among the static properties, our results for the masses are in very good agreement with previous results obtained with the same inter-quark interactions but within a more complicated Faddeev approach [15]. In some cases we even get lower, and thus better, masses. We have calculated mass densities and charge densities (charge form factors) finding that the corresponding mean square radii are again in good agreement with the Faddeev calculation of Ref. [15]. We have also evaluated magnetic moments. Being the total orbital angular momentum of the baryon $L = 0$, the magnetic moments come from the spin contributions alone. With the exception of Ξ^0_{bc}, Ξ^+_{bc} and Ω^0_{bc} we agree perfectly with the Faddeev calculation in Ref. [15]. For the magnetic moments of Ξ^0_{bc}, Ξ^+_{bc} and Ω^0_{bc} the discrepancies between the two calculation are very large. The origin might be attributed to the presence of a non–negligible $S_h = 0$ component in the wave functions of Ref. [15]. In our case we have $S_h = 1$ which we think is a good approximation based on the infinite heavy quark mass limit. This assertion seems to be corroborated by the results obtained in the relativistic calculation of Ref. [27], at least for the Ξ^0_{bc} and Ω^0_{bc} cases.

We have used our simple wave functions to study the semileptonic decay of doubly $\Xi(J = 1/2)$ and $\Omega(J = 1/2)$ baryons. We have worked in the spectator approximation with one–body currents alone. In the $m_b = m_c$ case and for $B = B'$ baryons we have checked that our model satisfies baryon number conservation. On the other hand we have a small vector current violation by an amount given by the binding energy over the mass of the baryon. With this model we have evaluated form factors, asymmetry parameters, differential decay widths and total decay widths. Our results for the latter are in reasonable agreement with the ones obtained in Ref. [32] using a relativistic quark model in the quark–diquark approximation, while they are much smaller than the ones obtained in Ref. [34] by means of the Bethe–Salpeter equation applied to a quark-diquark system.

For the weak form factors the results exhibit an apparent SU(3) symmetry when going from Ξ to Ω baryons. This is due to the fact that we have two heavy quarks and the light one acts as a spectator in the weak transition. This apparent symmetry appears also in the decay widths and asymmetry parameters. On the other hand SU(3) violating effects are clearly visible in some static quantities like the charge form factors and radii, the light quark mass densities , and the magnetic moments, that depend strongly on the light quark charge and/or mass.

Acknowledgments

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APPENDIX A: VARIATIONAL WAVE FUNCTION PARAMETERS

In Tables XI (doubly heavy Ξ baryons) and XII (doubly heavy Ω baryons) we give the variational parameters of the orbital wave functions evaluated with the different quark–quark interactions analyzed in this work.

		b_1	d_1	a_2	b_2	d_2	a_3	b_3	d_3	a_4	b_4	d_4	α_{h_1}	α_{h_2}
Ξ_{cc}	AL1	1.78	0.37	0.38	1.04	0.28	0.57	1.24	0.66	1.36	1.56	0.08	0.44	0.44
	AL2	2.48	0.09	1.25	1.50	0.01	0.34	1.55	1.46	2.20	2.21	1.82	0.45	0.45
	AP1	2.14	0.14	0.83	1.45	-0.01	0.67	1.95	1.77	2.69	2.98	1.34	0.40	0.40
	AP2	1.86	0.11	0.48	1.25	0.14	0.62	1.62	1.17	1.92	2.20	1.26	0.41	0.41
	BD	1.71	0.18	0.37	1.23	0.16	0.85	1.59	0.96	1.46	2.07	1.22	0.44	0.44
Ξ^*_{cc}	AL1	1.26	0.40	-0.03	0.82	-0.30	0.55	0.89	0.54	0.76	1.66	0.15	0.83	0.83
	AL2	1.18	0.53	-0.02	0.65	-0.36	0.58	0.85	0.58	1.29	1.71	0.12	0.83	0.83
	AP1	1.93	0.17	0.66	1.37	0.00	0.67	2.29	1.56	1.66	2.99	1.28	0.76	0.76
	AP2	1.18	0.46	-0.02	0.64	-0.38	0.43	0.79	0.65	1.59	1.73	0.12	0.75	0.75
	BD	1.23	0.31	-0.01	0.72	-0.22	0.53	0.97	0.63	1.22	1.79	0.17	0.73	0.73
Ξ_{bb}	AL1	2.37	0.23	1.26	1.31	0.85	1.47	1.93	1.04	1.17	2.09	2.13	0.39	0.39
	AL2	2.35	1.81	1.54	1.57	1.41	2.03	2.11	0.27	0.69	1.85	2.20	0.39	0.39
	AP1	2.66	1.52	1.57	1.48	1.42	1.99	1.94	0.37	0.85	1.77	2.31	0.35	0.35
	AP2	2.22	1.93	1.63	1.53	1.59	2.01	2.23	0.19	0.69	1.82	2.08	0.36	0.36
	BD	2.74	1.35	1.42	1.42	1.20	2.04	1.90	0.43	0.85	1.80	2.54	0.40	0.40
Ξ^*_{bb}	AL1	3.17	0.15	1.04	1.89	0.24	0.14	1.47	1.79	2.25	2.84	1.47	0.54	0.54
	AL2	2.82	0.11	0.56	1.56	0.39	0.57	2.14	1.47	1.47	2.80	1.89	0.52	0.52
	AP1	2.69	0.18	0.88	1.73	0.33	0.75	2.33	1.80	1.90	3.14	1.76	0.48	0.48
	AP2	2.78	0.08	0.41	1.60	0.33	0.71	2.05	1.45	1.56	2.87	1.91	0.49	0.49
	BD	3.40	0.13	0.19	2.74	-0.15	-0.07	2.13	2.04	2.64	3.85	3.02	0.51	0.51
Ξ'_{bc}	AL1	2.24	0.21	0.39	1.73	0.01	-0.01	2.18	-0.78	1.62	0.77	1.84	0.01	1.51
	AL2	2.40	0.12	0.28	1.67	-0.06	-0.01	2.15	-0.82	1.29	0.87	1.49	0.01	1.53
	AP1	4.81	-0.01	2.78	2.03	0.05	-0.15	3.45	0.52	2.31	0.91	1.09	0.01	1.37
	AP2	2.85	0.05	0.98	1.91	-0.03	-0.15	2.66	-0.19	0.68	0.99	1.55	0.02	1.34
	BD	2.60	0.14	0.58	1.62	0.05	-0.04	2.47	-0.66	1.65	0.94	1.28	-0.03	1.54
Ξ_{bc}	AL1	2.82	0.15	0.46	2.37	-0.15	-0.09	1.06	-0.37	1.31	0.70	1.35	-0.02	1.09
	AL2	2.97	0.08	0.57	2.14	-0.11	-0.05	1.15	-0.40	0.92	0.72	1.39	-0.06	1.20
	AP1	2.91	0.10	0.56	2.16	-0.10	-0.08	1.13	-0.50	1.21	0.72	1.25	-0.06	1.05
	AP2	4.75	-0.02	1.98	2.74	0.06	0.38	1.47	-0.16	3.03	2.13	-0.04	-0.07	1.09
	BD	2.64	0.21	0.48	2.38	-0.08	-0.12	1.02	-0.57	1.25	0.56	1.74	-0.07	1.20
Ξ^*_{bc}	AL1	1.85	0.21	-0.19	2.20	0.10	-0.02	1.26	-0.66	1.27	0.70	1.71	0.07	1.72
	AL2	1.93	0.13	-0.14	2.25	-0.09	-0.05	1.10	-0.56	0.92	0.62	1.67	0.06	1.74
	AP1	1.93	0.17	0.06	2.54	0.17	-0.10	0.97	-0.57	1.07	0.55	1.76	0.06	1.56
	AP2	2.16	0.07	-0.12	1.91	-0.08	-0.09	1.23	-0.78	0.84	0.66	1.14	0.07	1.48
	BD	2.79	0.12	0.55	2.14	-0.05	-0.03	1.65	-0.83	2.08	0.79	1.39	-0.01	1.70

TABLE XI: Variational parameters of the three body orbital wave function for doubly heavy Ξ baryons. $a's$ are dimensionless, $d's$ have dimensions of fm and $b's$ and $\alpha's$ have dimensions of fm^{-1}. Whenever the two heavy quark are different the suffix h_1 stands for a b quark and h_2 stands for a c quark.

		b_1	d_1	a_2	b_2	d_2	a_3	b_3	d_3	a_4	b_4	d_4	α_{h_1}	α_{h_2}
Ω_{cc}	AL1	1.60	0.29	0.30	1.29	1.20	1.70	1.27	0.25	0.94	0.85	1.33	0.53	0.53
	AL2	1.21	0.17	0.25	1.35	1.22	2.04	1.66	0.22	0.69	0.98	1.63	0.48	0.48
	AP1	2.30	0.11	0.48	1.70	1.26	1.72	1.36	0.12	0.08	0.56	2.55	0.47	0.47
	AP2	1.63	-0.24	0.62	1.75	2.01	2.86	2.16	0.10	0.16	1.35	2.56	0.49	0.49
	BD	1.34	0.09	0.37	1.46	1.30	1.91	1.69	0.28	0.73	1.17	1.55	0.47	0.47
Ω_{cc}^*	AL1	1.29	0.16	-0.01	0.90	-0.49	0.96	0.99	0.75	0.67	2.05	0.13	0.95	0.95
	AL2	1.41	0.10	0.01	0.72	-0.47	0.99	1.00	0.77	0.71	2.00	0.15	0.89	0.89
	AP1	1.81	0.10	0.15	1.69	-0.49	1.35	1.06	0.86	-0.08	2.66	0.41	0.94	0.94
	AP2	2.32	0.04	0.22	1.97	-0.29	1.33	1.07	0.47	-0.65	2.31	1.70	0.92	0.92
	BD	1.66	0.15	0.16	1.41	-0.27	1.14	1.26	0.80	0.30	2.21	0.71	0.73	0.73
Ω_{bb}	AL1	1.28	0.91	-0.07	0.71	1.05	1.86	2.32	0.26	1.38	0.99	1.27	0.51	0.51
	AL2	1.20	0.82	-0.04	0.65	1.28	1.78	2.56	0.15	1.22	1.03	1.41	0.45	0.45
	AP1	1.18	0.90	-0.09	0.72	1.24	1.70	2.48	0.20	1.43	1.12	1.12	0.44	0.44
	AP2	2.00	0.03	0.22	0.58	1.70	2.81	3.22	0.06	-0.63	0.42	3.32	0.47	0.47
	BD	1.47	0.58	-0.09	0.79	1.16	1.68	2.48	0.25	1.25	1.00	1.52	0.46	0.46
Ω_{bb}^*	AL1	1.14	1.01	0.01	1.03	0.44	3.05	2.03	0.36	1.20	2.48	1.37	0.68	0.68
	AL2	0.97	1.30	0.08	0.98	0.75	2.79	2.36	0.19	1.21	2.40	1.38	0.63	0.63
	AP1	1.05	1.16	0.11	1.29	0.56	3.25	2.16	0.29	0.86	2.54	1.31	0.63	0.63
	AP2	0.94	1.30	0.33	1.11	0.76	3.39	2.55	0.11	0.79	2.45	1.75	0.65	0.65
	BD	1.52	1.12	0.19	1.15	0.74	2.91	2.05	0.37	1.29	2.44	1.41	0.57	0.57
Ω_{bc}'	AL1	2.20	0.32	1.00	1.92	0.05	-0.09	2.89	-0.24	1.79	0.75	1.87	-0.14	2.42
	AL2	2.21	0.22	0.76	2.03	-0.07	-0.16	2.94	-0.30	1.56	0.78	1.92	-0.18	2.34
	AP1	5.34	0.04	2.24	2.83	0.01	-1.34	4.37	0.29	0.64	2.14	-0.28	-0.17	2.35
	AP2	3.33	0.01	1.71	2.14	-0.04	-0.36	3.00	-0.20	1.23	0.71	1.88	-0.17	2.34
	BD	3.06	0.14	0.83	1.94	0.18	0.44	2.16	-0.11	1.67	0.75	1.78	-0.24	2.26
Ω_{bc}	AL1	2.15	0.14	-0.04	1.22	-0.32	-0.02	2.33	-1.04	1.10	0.84	0.98	-0.20	1.84
	AL2	2.03	0.15	0.03	1.39	-0.36	-0.01	2.37	-0.81	1.22	0.75	1.78	-0.25	1.84
	AP1	1.89	0.18	-0.02	1.49	-0.40	0.04	2.10	-0.61	1.01	0.81	1.90	-0.24	1.78
	AP2	2.34	0.02	0.51	4.50	-0.01	0.53	1.82	-0.08	3.95	2.69	5.47	-0.23	1.78
	BD	2.17	0.18	0.15	1.50	-0.06	-0.01	2.46	-0.89	1.41	0.75	1.97	-0.27	1.85
Ω_{bc}^*	AL1	2.59	0.10	1.01	1.90	-0.01	-0.27	2.88	-0.13	0.87	0.51	2.88	-0.06	2.54
	AL2	2.69	0.11	1.15	1.65	0.10	0.08	2.80	-0.07	1.28	0.64	2.90	-0.13	2.51
	AP1	2.59	0.14	0.96	1.88	-0.01	-0.08	2.82	-0.23	1.15	0.52	3.09	-0.10	2.59
	AP2	3.80	-0.01	1.75	1.81	0.03	0.69	2.31	0.03	2.48	0.56	2.94	-0.11	2.60
	BD	2.67	0.12	1.25	1.88	0.06	-0.21	2.97	-0.11	1.15	0.52	3.16	-0.20	2.35

TABLE XII: Same as Table XI for doubly heavy Ω baryons.

APPENDIX B: FORM FACTORS IN TERMS OF THE $\mathcal{I}^{B'B}(|\vec{q}\,|)$ AND $\mathcal{K}^{B'B}(|\vec{q}\,|)$ INTEGRALS

In this appendix we relate the vector F_1, F_2, F_3 and axial G_1, G_2, G_3 form factors, that we evaluate in the center of mass of the decaying baryon, to the integrals $\mathcal{I}^{B'B}(|\vec{q}\,|)$, $\mathcal{K}^{B'B}(|\vec{q}\,|)$ defined in Eq.(36). To simplify the expressions it is convenient to introduce

$$\begin{aligned}
\widehat{F}_j(|\vec{q}\,|) &= \sqrt{\frac{E_{B'}(-\vec{q}\,)+m_{B'}}{2E_{B'}(-\vec{q}\,)}}\sqrt{\frac{2E_c(\vec{q}\,)}{E_c(\vec{q}\,)+m_c}}\;F_j(|\vec{q}\,|) \quad , \; j=1,2,3 \\
\widehat{G}_j(|\vec{q}\,|) &= \sqrt{\frac{E_{B'}(-\vec{q}\,)+m_{B'}}{2E_{B'}(-\vec{q}\,)}}\sqrt{\frac{2E_c(\vec{q}\,)}{E_c(\vec{q}\,)+m_c}}\;G_j(|\vec{q}\,|) \quad , \; j=1,2,3
\end{aligned} \tag{B1}$$

- Cases $S_h=1$, $S_h'=0$ or $S_h=0$, $S_h'=1$:

$$\frac{\widehat{F}_1(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} = -\frac{2}{\sqrt{3}}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right)$$

$$
\begin{aligned}
\frac{\widehat{F}_1(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} + \frac{\widehat{F}_3(|\vec{q}\,|)}{m_{B'}} &= 0 \\
\widehat{F}_1(|\vec{q}\,|)+\widehat{F}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{F}_3(|\vec{q}\,|) &= 0
\end{aligned}
\tag{B2}
$$

where $E_c(\vec{q}\,)=\sqrt{m_c^2+\vec{q}^{\,2}}$. From the above expressions we have that $F_2=F_3$.

$$
\begin{aligned}
-\,\widehat{G}_1(|\vec{q}\,|) &= \frac{2}{\sqrt{3}}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right) \\
\left(-\widehat{G}_1(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\widehat{G}_3(|\vec{q}\,|)}{m_{B'}(E_{B'}(-\vec{q}\,)+m_{B'})}\right) &= \frac{2}{\sqrt{3}}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right) \\
\frac{-\widehat{G}_1(|\vec{q}\,|)+\widehat{G}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{G}_3(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} &= \frac{2}{\sqrt{3}}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right)
\end{aligned}
\tag{B3}
$$

- Case $S_h=1$, $S'_h=1$

$$
\begin{aligned}
\frac{\widehat{F}_1(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} &= \frac{4}{3}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right) \\
\frac{\widehat{F}_1(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}}+\frac{\widehat{F}_3(|\vec{q}\,|)}{m_{B'}} &= 2\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right) \\
\widehat{F}_1(|\vec{q}\,|)+\widehat{F}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{F}_3(|\vec{q}\,|) &= 2\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right)
\end{aligned}
\tag{B4}
$$

$$
\begin{aligned}
\widehat{G}_1(|\vec{q}\,|) &= \frac{4}{3}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right) \\
\left(-\widehat{G}_1(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\widehat{G}_3(|\vec{q}\,|)}{m_{B'}(E_{B'}(-\vec{q}\,)+m_{B'})}\right) &= -\frac{4}{3}\left(\mathcal{I}^{B'B}(|\vec{q}\,|)+\frac{|\vec{q}\,|^2\,\mathcal{K}^{B'B}(|\vec{q}\,|)}{2\,(E_c(\vec{q}\,)+m_c)}\left(\frac{m_c}{E_c^2(\vec{q}\,)}-\frac{1}{m_b}\right)\right) \\
\frac{-\widehat{G}_1(|\vec{q}\,|)+\widehat{G}_2(|\vec{q}\,|)+\frac{E_{B'}(-\vec{q}\,)}{m_{B'}}\,\widehat{G}_3(|\vec{q}\,|)}{E_{B'}(-\vec{q}\,)+m_{B'}} &= -\frac{4}{3}\left(\frac{\mathcal{I}^{B'B}(|\vec{q}\,|)}{E_c(\vec{q}\,)+m_c}-\frac{\mathcal{K}^{B'B}(|\vec{q}\,|)}{2}\left(\frac{m_c}{E_c^2(\vec{q}\,)}+\frac{1}{m_b}\right)\right)
\end{aligned}
\tag{B5}
$$

APPENDIX C: $\mathcal{I}^{B'B}(|\vec{q}\,|)$ AND $\mathcal{K}^{B'B}(|\vec{q}\,|)$ INTEGRALS

To evaluate $\mathcal{I}^{B'B}(|\vec{q}\,|)$ and $\mathcal{K}^{B'B}(|\vec{q}\,|)$ we use a partial wave expansion of the orbital wave functions:

$$
\begin{aligned}
\Psi^{B}_{b\,h_2}(r_1,r_2,r_{12}) &= \sum_{l=0}^{\infty} f^{B}_l(r_1,r_2)P_l(\mu) \\
\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12}) &= \sum_{l=0}^{\infty} f^{B'}_l(r_1,r_2)P_l(\mu)
\end{aligned}
\tag{C1}
$$

where μ is the cosine of the angle made by $\vec{r}_1$ and $\vec{r}_2$ and $P_l(\mu)$ is a Legendre polynomial of rank l. The radial functions $f_l^B(r_1,r_2)$, $f_l^{B'}(r_1,r_2)$, are evaluated as

$$\begin{aligned}
f_l^B(r_1,r_2) &= \frac{2l+1}{2}\int_{-1}^{+1} d\mu\; P_l(\mu)\,\Psi^B_{b\,h_2}(r_1,r_2,r_{12}) \\
f_l^{B'}(r_1,r_2) &= \frac{2l+1}{2}\int_{-1}^{+1} d\mu\; P_l(\mu)\,\Psi^{B'}_{c\,h_2}(r_1,r_2,r_{12})
\end{aligned} \tag{C2}$$

In terms of those we have

- $\mathcal{I}^{B'B}(|\vec{q}\,|)$

$$\begin{aligned}
\mathcal{I}^{B'B}(|\vec{q}\,|) &= 16\pi^2\sum_l\sum_{l'}\sum_{l''}(ll'l''\,|\,000)^2\int_0^\infty d\,r_1\,r_1^2\;j_{l''}(\frac{m_{h_2}+m_q}{\overline{M}'}|\vec{q}\,|r_1)\int_0^\infty d\,r_2\,r_2^2\;j_{l''}(\frac{m_{h_2}}{\overline{M}'}|\vec{q}\,|r_2) \\
&\quad\times f_{l'}^{B'}(r_1,r_2)\,f_l^B(r_1,r_2)
\end{aligned} \tag{C3}$$

  with $j's$ being spherical Bessel functions.

- $\mathcal{K}^{B'B}(|\vec{q}\,|)$

$$\begin{aligned}
\mathcal{K}^{B'B}(|\vec{q}\,|) &= -\frac{16\pi^2}{\sqrt{3}\,|\vec{q}\,|}\sum_l\sum_{l'}\sum_{l''}\sum_{l'''}\sum_L(-1)^{(l''+l'''+1)/2}\sqrt{(2L+1)(2l''+1)(2l'''+1)} \\
&\quad\times(ll'l''\,|\,000)\;(l''l'''1\,|\,000)\;(l'Ll'''\,|\,000)\;W(l''l'1L:ll''') \\
&\quad\times\int_0^\infty d\,r_1\,r_1^2\;j_{l'''}(\frac{m_{h_2}+m_q}{\overline{M}'}|\vec{q}\,|r_1)\int_0^\infty d\,r_2\,r_2^2\;j_{l''}(\frac{m_{h_2}}{\overline{M}'}|\vec{q}\,|r_2)\;f_{l'}^{B'}(r_1,r_2)\,\Omega_L\,f_l^B(r_1,r_2)
\end{aligned} \tag{C4}$$

  with $W(l''l'1L;l,l''')$ being a Racah coefficient and Ω_L the differential operator[11]

$$\begin{aligned}
\Omega_{L=l+1} &= -\sqrt{\frac{l+1}{2l+1}}\left(\frac{\partial}{\partial r_1}-\frac{l}{r_1}\right) \\
\Omega_{L=l-1} &= \sqrt{\frac{l}{2l+1}}\left(\frac{\partial}{\partial r_1}+\frac{l+1}{r_1}\right)
\end{aligned} \tag{C5}$$

For the actual evaluation we restrict the l, l' values to l, $l' = 0,\cdots,6$

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[11] Note that the Racah and Clebsch-Gordan coefficients restrict L to the two possible values $L = l \pm 1$.

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